Structural Form

Demand : $q_t = \alpha_1 p_t + \alpha_2 x_t + \epsilon_{dt}$

Supply: $q_t = \beta_1 p_t + \epsilon_{st}$

Reduced Form

$$p_t = \frac{\alpha_2 x_t}{\beta_1 - \alpha_1} + \frac{\varepsilon_{dt} - \varepsilon_{st}}{\beta_1 - \alpha_1} = \pi_1 x_t + v_{1t}$$

$$q_t = \frac{\beta_1 \alpha_2 x_t}{\beta_1 - \alpha_1} + \frac{\beta_1 \varepsilon_{dt} - \alpha_1 \varepsilon_{st}}{\beta_1 - \alpha_1} = \pi_2 x_t + v_{2t}$$

$$\beta_1 = \frac{\pi_2}{\pi_1} \qquad \alpha_1 = ? \qquad \alpha_2 = ?$$

Structural Form

Demand : $q^d_t = \alpha_1 p_t + \alpha_2 x_t + \epsilon_{dt}$

Supply: $q^s_t = \beta_1 p_t + \epsilon_{st}$

Linear Combination of Demand and Supply

$$q_t = \lambda q^d_t + (1-\lambda) q^s_t$$

$$= [\lambda\alpha_1 + (1-\lambda)\beta_1] p_t + \lambda\alpha_2 x_t + [\lambda\epsilon_{dt} + (1-\lambda)\epsilon_{st}]$$

Structural Form

Demand : $q_t = \alpha_1 p_t + \alpha_2 x_t + \epsilon_{dt}$

Supply: $q_t = \beta_1 p_t + \beta_2 w_t + \epsilon_{st}$

Reduced Form

$$p_t = \frac{\alpha_2 x_t}{\beta_1 - \alpha_1} + \frac{\beta_2 w_t}{\beta_1 - \alpha_1} + \frac{\varepsilon_{dt} - \varepsilon_{st}}{\beta_1 - \alpha_1} = \pi_1 x_t + \pi_3 w_t + v_{1t}$$

$$q_t = \frac{\beta_1 \alpha_2 x_t}{\beta_1 - \alpha_1} + \frac{\alpha_1 \beta_2 w_t}{\beta_1 - \alpha_1} + \frac{\beta_1 \varepsilon_{dt} - \alpha_1 \varepsilon_{st}}{\beta_1 - \alpha_1} = \pi_2 x_t + \pi_4 w_t + v_{2t}$$

$$\beta_1 = \frac{\pi_2}{\pi_1} \quad \alpha_1 = \frac{\pi_4}{\pi_3} \quad \alpha_2 = \pi_1(\beta_1 - \alpha_1) \quad \beta_2 = \pi_3(\beta_1 - \alpha_1)$$

Structural Form

Demand : $q_t = \alpha_1 p_t + \alpha_2 x_t + \epsilon_{dt}$

Supply: $q_t = \beta_1 p_t + \beta_2 x_t + \epsilon_{st}$

Reduced Form

$$p_t = \frac{\alpha_2 x_t}{\beta_1 - \alpha_1} + \frac{\beta_2 x_t}{\beta_1 - \alpha_1} + \frac{\varepsilon_{dt} - \varepsilon_{st}}{\beta_1 - \alpha_1} = \pi_1 x_t + v_{1t}$$

$$q_t = \frac{\beta_1 \alpha_2 x_t}{\beta_1 - \alpha_1} + \frac{\alpha_1 \beta_2 x_t}{\beta_1 - \alpha_1} + \frac{\beta_1 \varepsilon_{dt} - \alpha_1 \varepsilon_{st}}{\beta_1 - \alpha_1} = \pi_2 x_t + v_{2t}$$

$$\beta_1 = ? \quad \alpha_1 = ? \quad \alpha_2 = ? \quad \beta_2 = ?$$

Structural Form

Demand : $q_t = \alpha_1 p_t + \epsilon_{dt}$

Supply: $q_t = \beta_1 p_t + \epsilon_{st}$

Reduced Form

$$p_t = \frac{\varepsilon_{dt} - \varepsilon_{st}}{\beta_1 - \alpha_1} = v_{1t}$$

$$q_t = \frac{\beta_1 \varepsilon_{dt} - \alpha_1 \varepsilon_{st}}{\beta_1 - \alpha_1} = v_{2t}$$

$$\beta_1 = ? \quad \alpha_1 = ? \quad \alpha_2 = ? \quad \beta_2 = ?$$

Structural Form

Demand : $q_t = \alpha_1 p_t + \alpha_2 x_t + \alpha_3 p'_t + \epsilon_{dt}$

Supply: $q_t = \beta_1 p_t + \epsilon_{st}$

Reduced Form

$$p_t = \frac{\alpha_2 x_t}{\beta_1 - \alpha_1} + \frac{\alpha_3 p'_t}{\beta_1 - \alpha_1} + \frac{\varepsilon_{dt} - \varepsilon_{st}}{\beta_1 - \alpha_1} = \pi_1 x_t + \pi_3 p'_t + v_{1t}$$

$$q_t = \frac{\beta_1 \alpha_2 x_t}{\beta_1 - \alpha_1} + \frac{\beta_1 \alpha_3 p'_t}{\beta_1 - \alpha_1} + \frac{\beta_1 \varepsilon_{dt} - \alpha_1 \varepsilon_{st}}{\beta_1 - \alpha_1} = \pi_2 x_t + \pi_4 p'_t + v_{2t}$$

$$\beta_1 = \frac{\pi_2}{\pi_1} \quad \beta_1 = \frac{\pi_4}{\pi_3} \quad \alpha_1 = ? \quad \alpha_2 = ? \quad \alpha_3 = ?$$

Linear Combination of equations (1) and (4)

(4) - $[\theta_{13}/\theta_2]$ (1) =

$$y_1 - [\theta_{13}/\theta_2]\, y_2 + \theta_{14}\, y_4 + [1 - \theta_3\, \theta_{13}/\theta_2\,]\, x_1 = 0$$

Equation (2):

$$y_1 + \theta_5\, y_2 + \theta_6\, y_4 + \theta_7\, x_1 = 0$$

RANK CONDITION

$$\begin{array}{c} \begin{matrix} y_3 & x_2 & x_3 \end{matrix} \\ \begin{vmatrix} \theta_2 & 0 & 0 \\ \theta_9 & \theta_{10} & \theta_{11} \\ \theta_{13} & 0 & 0 \end{vmatrix} \end{array} \neq 0$$

Reduced Form

$$p_t = \frac{\alpha_2 x_t}{\beta_1 - \alpha_1} + \frac{\varepsilon_{dt} - \varepsilon_{st}}{\beta_1 - \alpha_1} = \pi_1 x_t + v_{1t}$$

$$q_t = \frac{\beta_1 \alpha_2 x_t}{\beta_1 - \alpha_1} + \frac{\beta_1 \varepsilon_{dt} - \alpha_1 \varepsilon_{st}}{\beta_1 - \alpha_1} = \pi_2 x_t + v_{2t}$$

$$\epsilon_{dt} = v_{2t} - \alpha_1 v_{1t} \qquad \epsilon_{st} = v_{2t} - \beta_1 v_{1t}$$

$$\begin{aligned} E\epsilon_{dt}\,\epsilon_{st} &= Ev_{2t}^2 + \alpha_1 \beta_1 v_{1t}^2 - (\alpha_1+\beta_1)E\, v_{1t}\, v_{2t} \\ &= \sigma_2^2 + \alpha_1 \beta_1 \sigma_1^2 - (\alpha_1+\beta_1)\sigma_{12} = 0 \end{aligned}$$

$$\alpha_1 = \frac{\beta_1 \sigma_{12} - \sigma_1^2}{\beta_1 \sigma_1^2 - \sigma_{12}}$$