Answers to the Exercises -- Chapter 1

SECTION 1

1. a Sentence, official notation

```
~~~P
  |
 ~~P
  |
  ~P
  |
  P
```

b Sentence, informal notation

```
~Q→~R
   /\
~Q    ~R
 |     |
 Q     R
```

c Not a sentence; it is impossible to construct "~→"

d Not a sentence; the rules of formation do not allow you to enclose a negated formula in parentheses, only conditionals.

e Sentence, informal notation

```
(P→Q) → (R→~Q)
        /\
(P→Q)   (R→~Q)
  /\       /\
 P  Q     R  ~Q
              |
              Q
```

f Not a sentence; conditionals contained inside other conditionals must be surrounded by parentheses.

g Not a sentence; this is the same as sentence (f) with extra parentheses put on the outside; If (g) were a sentence, then (f) would be a sentence in informal notation; since (f) is not a sentence, (g) can't be either.

h Sentence; informal notation

```
(~S→R) → ((~R→S) → ~(~S→R))
          /\
~S→R        (~R→S) → ~(~S→R)
  /\                 /\
~S  R        ~R→S      ~(~S→R)
 |             /\          |
 S           ~R   S      ~S→R
              |            /\
              R          ~S   R
                          |
                          S
```

i Sentence; informal notation

```
P → (Q→P)
    /\
P     Q→P
        /\
       Q   P
```

SECTION 2

1. a $S \rightarrow R$, because "only if" immediately precedes the consequent
   b $R \rightarrow S$, because "provided that" is equivalent to "if", and "if" immediately precedes the antecedent
   c ~S, because "won't" means the same as "will not", the sentence only contains one negation indicator
   d $S \rightarrow R$, because "only if" immediately precedes the consequent
   e $R \rightarrow S$, because "given that" is equivalent to "if" and "if" immediately precedes the antecedent

2. a Susan will be late only **provided that** it rains
   Susan will be late only **if** it rains
   $S \rightarrow R$

   b Only **on condition that** it rains will Susan be late
   Only **if** it rains will Susan be late
   $S \rightarrow R$

   c Susan will be late only **in case** it rains
   Susan will be late only **if** it rains
   $S \rightarrow R$

   d Susan will be late only if it rains
   $S \rightarrow R$

   e It is not the case that Susan will be late
   ~S

SECTION 3

1. a *If Veronica doesn't leave William won't **either***
   *If Veronica doesn't leave William won't **leave***
   *If Veronica doesn't leave **then** William won't leave*
   $\sim V \rightarrow \sim W$

   b *William will leave if Yolanda does, **provided that** Veronica doesn't*
   William will leave if Yolanda does, if Veronica doesn't [leave]
   If Veronica doesn't [leave], then (William will leave if Yolanda does)
   If Veronica doesn't [leave], then (if Yolanda [leaves], then William will leave)
   $\sim V \rightarrow (Y \rightarrow W)$

   c *If Yolanda doesn't leave, then Veronica will leave only if William doesn't*
   $\sim Y \rightarrow (V \rightarrow \sim W)$

d *If Yolanda doesn't leave then Veronica will leave,* ***given that*** *William doesn't*
If Yolanda doesn't leave then Veronica will leave, if William doesn't
If William doesn't [leave], then (if Yolanda doesn't leave then Veronica will leave)
$\sim W \rightarrow (\sim Y \rightarrow V)$

2. a *William will leave if Veronica does*
If Veronica [leaves], then William will leave
$V \rightarrow W$

b *Veronica won't leave if William does*
If William [leaves], then Veronica won't leave
$W \rightarrow \sim V$

c *If Veronica leaves, then if William doesn't leave, Yolanda will leave*
If Veronica leaves, then $(\sim W \rightarrow Y)$
$V \rightarrow (\sim W \rightarrow Y)$

d *If Veronica doesn't leave if William doesn't, then Yolanda won't*
If (if William doesn't [leave], then Veronica doesn't leave), then Yolanda won't [leave]
$(\sim W \rightarrow \sim V) \rightarrow \sim Y$

e *William won't leave* ***provided that*** *Veronica doesn't leave*
William won't leave if Veronica doesn't leave
If Veronica doesn't leave, then William won't leave
$\sim V \rightarrow \sim W$

f *If William leaves, then if Veronica leaves* ***so will*** *Yolanda*
If William leaves, then (if Veronica leaves, then Yolanda will leave)
$W \rightarrow (V \rightarrow Y)$

g *William will leave only if if Veronica leaves then so will Yolanda*
William will leave only if (if Veronica leaves then so will Yolanda)
William will leave $\rightarrow$ *(if Veronica leaves then* ***so will*** *Yolanda)*
William will leave $\rightarrow$ *(if Veronica leaves then Yolanda will leave)*
$W \rightarrow (V \rightarrow Y)$

h *William will leave only if Veronica leaves, only* ***provided that*** *Yolanda will leave*
(William will leave only if Veronica leaves) only if Yolanda will leave
$(W \rightarrow V) \rightarrow Y$

SECTION 4

1. a None of the above; it might look like a modus tollens inference, but the second premise, Q, would have to first be changed to ~~Q by applying double negation; so while the argument is valid, it is not a one-step application of modus tollens.
b None of the above
c Double negation
d Modus ponens
e Modus tollens
f None of the above
g Modus ponens; the consequent of the conditional does not need to be an atomic sentence, it can be molecular as well.
h None of the above; it may look like a modus tollens inference, but the second premise is not actually the negation of the consequent of the first premise.

i Double negation and none of the above are both good answers; the conclusion can be inferred by double negation from the first premise, but since the second premise is not involved in that inference, the *whole argument* is not a double-negation inference

2. In all cases we can validly infer any sentence which results from putting '~~' in front of either premise by the rule double negation. Such results are not enumerated below.

a ~X may be inferred by modus ponens.
b X may be inferred by double negation; ~~W may be inferred by modus tollens.
c Nothing additional
d (R→X) may be inferred by modus ponens.
e Nothing additional; Modus tollens cannot be applied because the second premise is not actually the negation of the consequent of the first premise.
f (W→X) may be inferred from the first premise by double negation; this must be done before you apply modus tollens with the second premise, so you can't apply modus tollens in *one step* to get ~W.
g Nothing additional; if you apply double negation to the second premise you can then apply modus tollens as a *second step* to get ~W.
h Nothing additional
i (W→X) follows by double negation.

SECTION 5

1. Only errors are listed.

<u>First derivation</u>
Line 6 -- line 6 is not available at line 6; derivation can be corrected by writing "5 dn".

<u>Second derivation</u>
Line 3 -- when justifying writing a premise, no line citation is given
Line 4 -- the sentence on line 2 is not the negation of the consequent of the sentence on line 3; in this case we would need to first apply dn to line 2 as an intermediate step. Then we could apply mt with line 3, which would result in ~~P. P could then be inferred on the following line by dn.
Line 5 -- two lines must be cited with mp; the sentence inferred does follow from line 4 together with the first premise, but the first premise must be cited somehow.
Line 7 -- "5 6 mt" would result in ~R rather than ~~R.
Line 9 -- "5 8 mp" is OK, but the derivation is not done; we set out to show ~R, but line 9 displays ~Q, so we cannot conclude the derivation at this point and so it is incorrect to write "dd" to mark the conclusion of the derivation of ~R.

<u>Third derivation</u>
Line 3 -- "~S" is not one of the premises.
Line 7 -- neither line 5 nor line 6 is a conditional, so mt cannot possibly apply to that pair of lines.

2. In each case the derivation displayed does not represent the only possible derivation; many alternate, equally correct derivations can be given.

P
Q → ~P
R → Q
∴ ~R

1.	~~Show~~ ~R	
2.	P	pr
3.	~~P	2 dn
4.	Q → ~P	pr
5.	~Q	3 4 mt
6.	R → Q	pr
7.	~R	5 6 mt
8.		7 dd

In this derivation and those below, the "dd" could occur at the end of the previous line.

W → ~(V→~Y)
X → (V→~Y)
V → Y
(V→Y) → X
∴ ~W

1.	~~Show~~ ~W	
2.	V → Y	pr
3.	(V→Y) → X	pr
4.	X	2 3 mp
5.	X → (V→~Y)	pr
6.	V→~Y	4 5 mp
7.	~~(V→~Y)	6 dn
8.	W → ~(V→~Y)	pr
9.	~W	7 8 mt
10.		9 dd

(W→Z) → (Z→W)
(Z→W) → ~X
P → X
~~P
∴ ~(W→Z)

1.	~~Show~~ ~(W→Z)	
2.	~~P	pr
3.	P	2 dn
4.	P → X	pr
5.	X	3 4 mp
6.	~~X	5 dn
7.	(Z→W) → ~X	pr
8.	~(Z→W)	6 7 mt
9.	(W→Z) → (Z→W)	pr
10.	~(W→Z)	8 9 mt
11.		10 dd

SECTION 6

1. Only errors are listed.

Derivation a
All correct

Derivation b
Line 3 -- Line 1 is not available at line 3 because *when line 3 is written*, line 1 is still an uncancelled show line.
Line 7 -- No problem with the use of cd to box and cancel, but mt cannot be applied to lines 5 and 6 because line 6 does not contain the negation of the consequent of line 5; you would have to add a line and apply double negation to line 6 first.

Derivation c
Line 2 -- You can only assume the antecedent of the conditional to be shown.
Line 3 -- While the sentence on line 3 does logically follow from line 2 and premise 1, you can't apply mp to line 2 alone.
Line 9 -- The application of dn to line 8 is OK, but you can't end a conditional derivation on a line that does not contain the consequent of the conditional you set out to show.

2. In each case the derivation displayed does not represent the only possible derivation; many alternate, equally correct derivations can be given.

a. $P \to (Q \to (R \to S))$
$\sim Q \to \sim R$
R
$\therefore\ P \to S$

1.	~~Show~~ $P \to S$	
2.	P	ass cd
3.	$P \to (Q \to (R \to S))$	pr
4.	$Q \to (R \to S)$	2 3 mp
5.	R	pr
6.	$\sim\sim R$	5 dn
7.	$\sim Q \to \sim R$	pr
8.	$\sim\sim Q$	6 7 mt
9.	Q	8 dn
10.	$R \to S$	4 9 mp
11.	S	5 10 mp
12.		11 cd

b. $Q \to \sim(R \to S)$
$P \to (R \to S)$
$\sim Q \to R$
$\therefore\ P \to S$

1.	~~Show~~ $P \to S$	
2.	P	ass cd
3.	$P \to (R \to S)$	pr
4.	$R \to S$	2 3 mp
5.	$\sim\sim(R \to S)$	4 dn
6.	$Q \to \sim(R \to S)$	pr
7.	$\sim Q$	5 6 mt
8.	$\sim Q \to R$	pr
9.	R	7 8 mp
10.	S	4 9 mp
11.		10 cd

c. $U \to (U \to V)$
$\sim R \to \sim(U \to V)$
$R \to \sim S$
$\therefore\ U \to \sim S$

1.	~~Show~~ $U \to \sim S$	
2.	U	ass cd
3.	$U \to (U \to V)$	pr
4.	$U \to V$	2 3 mp
5.	$\sim\sim(U \to V)$	4 dn
6.	$\sim R \to \sim(U \to V)$	pr
7.	$\sim\sim R$	5 6 mt
8.	R	7 dn
9.	$R \to \sim S$	pr
10.	$\sim S$	8 9 mp
11.		10 cd

3. a $P \to T$
$\sim X \to \sim T$
$\sim S \to \sim X$
$\therefore\ P \to S$

1.	~~Show~~ $P \to S$	
2.	P	ass cd
3.	$P \to T$	pr
4.	T	2 3 mp
5.	$\sim\sim T$	4 dn
6.	$\sim X \to \sim T$	pr
7.	$\sim\sim X$	5 6 mt
8.	$\sim S \to \sim X$	pr
9.	$\sim\sim S$	7 8 mt
10.	S	9 dn
11.		10 cd

b $T \to S$
$Y \to (S \to P)$
$P \to \sim X$
Y
$\therefore\ T \to \sim X$

1.	~~Show~~ $T \to \sim X$	
2.	T	ass cd
3.	$T \to S$	pr
4.	S	2 3 mp
5.	$Y \to (S \to P)$	pr
6.	Y	pr
7.	$S \to P$	5 6 mp
8.	P	4 7 mp
9.	$P \to \sim X$	pr
10.	$\sim X$	8 9 mp cd

c $S \to \sim T$
$\sim S \to \sim R$
$\sim R \to X$
$Y \to T$
$\therefore\ \sim X \to \sim Y$

1.	~~Show~~ $\sim X \rightarrow \sim Y$	
2.	$\sim X$	ass cd
3.	$\sim R \rightarrow X$	pr
4.	$\sim\sim R$	2 3 mt
5.	$\sim S \rightarrow \sim R$	pr
6.	$\sim\sim S$	4 5 mt
7.	S	6 dn
8.	$S \rightarrow \sim T$	pr
9.	$\sim T$	7 8 mp
10.	$Y \rightarrow T$	pr
11.	$\sim Y$	9 10 mt cd

SECTION 7

1. Only errors are listed.

<u>Derivation a</u>
All correct

<u>Derivation b</u>
Line 3 -- The sentence on line 3 is not a premise.
Line 4 -- Line 2 is not the negation of the consequent of line 3 so mt doesn't apply. You would first have to apply dn to line 2. Even in that case the result would be ~~S rather than ~S.
Line 5 -- "ass id" may only appear on the line immediately following a show line.
Line 7 -- The mt inference is OK, but 5 and 7 do not directly contradict so id is used incorrectly. The derivation could be concluded on line 7 with "4 id" since 4 and 7 contradict directly.

<u>Derivation c</u>
Line 2 -- The sentence on the line is not the negation (or the un-negation, for that matter) of the show line.
Line 4 -- There is no way to apply mt with lines 2 and 3.
Line 6 -- 2 is not the negation of the consequent of 5 (though it should have been).
Line 7 -- There is no way that you can apply mt with lines 5 and 6; 2 and 7 don't contradict directly, so it is premature to conclude with id.

2. In each case the derivation displayed does not represent the only possible derivation; many alternate, equally correct derivations can be given.

a. $\sim Q \rightarrow R$
$S \rightarrow \sim R$
$\sim S \rightarrow Q$
$\therefore$ Q

1.	~~Show~~ Q	
2.	$\sim Q$	ass id
3.	$\sim Q \rightarrow R$	pr
4.	R	2 3 mp
5.	$\sim\sim R$	4 dn
6.	$S \rightarrow \sim R$	pr
7.	$\sim S$	5 6 mt
8.	$\sim S \rightarrow Q$	pr
9.	Q	7 8 mp
10.		2 9 id

b. $(P \to Q) \to R$
$S \to (P \to Q)$
$\sim S \to R$
$\therefore$ R

1.	~~Show~~ R	
2.	$\sim R$	ass id
3.	$(P \to Q) \to R$	pr
4.	$\sim(P \to Q)$	2 3 mt
5.	$S \to (P \to Q)$	pr
6.	$\sim S$	4 5 mt
7.	$\sim S \to R$	pr
8.	R	6 7 mp
9.		2 8 id

c. $\sim P \to (R \to S)$
$(R \to S) \to T$
$\sim T$
$Q \to (R \to S)$
$\therefore \sim(P \to Q)$

1.	~~Show~~ $\sim(P \to Q)$	
2.	$P \to Q$	ass id
3.	$\sim T$	pr
4.	$(R \to S) \to T$	pr
5.	$\sim(R \to S)$	3 4 mt
6.	$Q \to (R \to S)$	pr
7.	$\sim Q$	5 6 mt
8.	$\sim P$	2 8 mt
9.	$\sim P \to (R \to S)$	pr
10.	$R \to S$	8 9 mp
11.		5 10 id

SECTION 8

1. Only errors are listed.

<u>Derivation a</u>
All correct

<u>Derivation b</u>
Line 8 -- At this point in the derivation, line 5 is still an un-cancelled show line so line 4 can't be cited to conclude the sub-derivation; you could, however, use the rule **r** (repetition) to repeat line 4 within the sub-derivation and then use the repeated line to conclude the sub-derivation with id.

<u>Derivation c</u>
Line 6 -- Line 6 is not available on line 6. The problem would b resolved if we put 5 for 6.
Line 13 -- Strictly speaking there is no error here, but it was un-necessary to repeat line 2 in order to apply id; we could have just cited "2 12 id" because line 2 is not separated from line 13 by any un-cancelled show lines (only by cancelled ones).

2. In each case the derivation displayed does not represent the only possible derivation; many alternate, equally correct derivations can be given.

a. $P \to (Q \to R)$

$S \to Q$

$\therefore\ S \to (P \to R)$

1.	~~Show~~ $S \to (P \to R)$	
2.	S	ass cd
3.	~~Show~~ $P \to R$	
4.	P	ass cd
5.	$P \to (Q \to R)$	pr
6.	$Q \to R$	4 5 mp
7.	$S \to Q$	pr
8.	Q	2 7 mp
9.	R	6 8 mp
10.		9 cd
11		3 cd

b. $(P \to Q) \to Q$

$P \to R$

$Q \to {\sim}Q$

$\therefore\ {\sim}(R \to Q)$

1.	~~Show~~ ${\sim}(R \to Q)$	
2.	$R \to Q$	ass id
3.	~~Show~~ $P \to Q$	
4.	P	ass cd
5.	$P \to R$	pr
6.	R	4 5 mp
7.	Q	2 6 mp cd
8.	$(P \to Q) \to Q$	pr
9.	Q	3 8 mp
10.	$Q \to {\sim}Q$	pr
11.	${\sim}Q$	9 10 mp 9 id

c. $(U \to V) \to (W \to X)$

$U \to Z$

${\sim}V \to {\sim}Z$

$X \to Z$

$\therefore\ W \to Z$

1.	~~Show~~ $W \to Z$	
2.	W	ass cd
3.	~~Show~~ $U \to V$	
4.	U	ass cd
5.	$U \to Z$	pr
6.	Z	4 5 mp
7.	${\sim}{\sim}Z$	6 dn
8.	${\sim}V \to {\sim}Z$	pr
9.	${\sim}{\sim}V$	7 8 mt
10.	V	9 dn cd
11.	$(U \to V) \to (W \to X)$	pr
12.	$W \to X$	3 11 mp
13.	X	2 12 mp
14.	$X \to Z$	pr
15.	Z	13 14 mp cd

SECTION 9

1\. a $P \to R$
$Q \to \sim R$
$\sim Q \to Q$
$\therefore P \to Q$

1.	~~Show~~ $P \to Q$	
2.	P	ass cd
3.	$P \to R$	pr
4.	R	2 3 mp
5.	$\sim\sim R$	4 dn
6.	$Q \to \sim R$	pr
7.	$\sim Q$	5 6 mt
8.	$\sim Q \to Q$	pr
9.	Q	7 8 mp cd

The only change was to conclude with cd instead of id.

b $Q \to U$
$Q \to \sim U$
$R \to Q$
R
$\therefore P$

1.	~~Show~~ P	
2.	$\sim P$	ass id
3.	R	pr
4.	$R \to Q$	pr
5.	Q	3 4 mp
6.	$Q \to U$	pr
7.	U	5 6 mp
8.	$Q \to \sim U$	pr
9.	$\sim U$	5 8 mp 7 id

The only change was to add an assumption for id.

c $U \to (V \to W)$
$X \to U$
$\sim X \to W$
$\therefore V \to W$

1.	~~Show~~ $V \to W$	
2.	$\sim(V \to W)$	ass id
3.	$U \to (V \to W)$	pr
4.	$\sim U$	2 3 mt
5.	$X \to U$	pr
6.	$\sim X$	4 5 mt
7.	$\sim X \to W$	pr
8.	W	6 7 mp
9.	~~Show~~ $V \to W$	
10.	V	ass cd
11.	W	8 r cd
12.		2 9 id

Only added lines 9-12.

2. a

$P \to R$
$Q \to {\sim}R$
${\sim}Q \to Q$
$\therefore\ P \to Q$

1.	~~Show~~ $P \to Q$	
2.	P	ass cd
3.	R	2 pr1 mp
4.	${\sim}{\sim}R$	3 dn
5.	${\sim}Q$	4 pr2 mt
6.	Q	5 pr3 mp cd

b

$Q \to U$
$Q \to {\sim}U$
$R \to Q$
R
$\therefore\ P$

1.	~~Show~~ P	
2.	${\sim}P$	ass id
3.	Q	pr4 pr3 mp
4.	U	3 pr1 mp
5.	${\sim}U$	3 pr2 mp 4 id

c

$U \to (V \to W)$
$X \to U$
${\sim}X \to W$
$\therefore\ V \to W$

1.	~~Show~~ $V \to W$	
2.	${\sim}(V \to W)$	ass id
3.	${\sim}U$	2 pr1 mt
4.	${\sim}X$	3 pr2 mt
5.	W	4 pr3 mp cd

3

P
Q
$\therefore\ P \to Q$

1.	~~Show~~ $P \to Q$	
2.	Q	pr2 cd

P
${\sim}Q$
$\therefore\ {\sim}(P \to Q)$

1.	~~Show~~ ${\sim}(P \to Q)$	
2.	$P \to Q$	ass id
3.	Q	pr1 2 mp
4.	${\sim}Q$	pr2 3 id

~P
Q
∴ P → Q

1.	~~Show~~ P → Q	
2.	Q	pr2 cd

~P
~Q
∴ P → Q

1.	~~Show~~ P → Q	
2.	P	ass cd
3.	~P	pr1 2 id

SECTION 10

1. S
(R→S) → W
∴ W

1.	~~Show~~ W	
2.	~~Show~~ R → S	
3.	S	pr1 cd
4.	W	2 pr2 mp dd

2. P → (S→R)
P → (W→S)
W → P
∴ W → R

1.	~~Show~~ W → R	
2.	W	as cd
3.	P	2 pr3 mp
4.	S → R	3 pr1 mp
5.	W → S	3 pr2 mp
6.	S	2 5 mp
7.	R	4 6 mp cd

3. (P→Q) → S
S → T
~T → Q
∴ T

1.	~~Show~~ T	
2.	~T	ass id
3.	Q	2 pr3 mp
4.	~~Show~~ P→Q	
5.	Q	3 r cd
6.	S	4 pr1 mp
7.	T	6 pr2 mp 2 id

SECTION 11

Derivations for theorems are not given here.

SECTION 12

1. a $X \to \sim(Y \to Z)$

$\therefore$ $(Y \to Z) \to \sim X$

1.	~~Show~~ $(Y \to Z) \to \sim X$	
2.	$(X \to \sim(Y \to Z)) \to ((Y \to Z) \to \sim X)$	T14
3.	$(Y \to Z) \to \sim X$	2 pr1 mp dd

b $R \to (\sim P \to S)$

$R \to \sim P$

$\therefore$ $R \to S$

1.	~~Show~~ $R \to S$	
2.	$(R \to (\sim P \to S)) \to ((R \to \sim P) \to (R \to S))$	T6
3.	$(R \to \sim P) \to (R \to S)$	2 pr1 mp
4.	$R \to S$	3 pr2 mp dd

c $\sim(R \to (S \to T))$

$R \to P$

$P \to (Q \to (S \to T))$

$\therefore$ $\sim Q$

1.	~~Show~~ $\sim Q$	
2.	Q	ass id
3.	$\sim(R \to (S \to T)) \to R$	T21
4.	R	3 pr1 mp
5.	P	4 pr2 mp
6.	$Q \to (S \to T)$	5 pr3 mp
7.	$S \to T$	2 6 mp
8.	$\sim(R \to (S \to T)) \to \sim(S \to T)$	T22
9.	$\sim(S \to T)$	8 pr1 mp 7 id

d $Q \to R$

$R \to S$

$\therefore$ $Q \to S$

1.	~~Show~~ $Q \to S$	
2.	$(Q \to R) \to ((R \to S) \to (Q \to S))$	T4
3.	$(R \to S) \to (Q \to S)$	2 pr1 mp
4.	$Q \to S$	3 pr2 mp dd