Answers to the Exercises -- Chapter 2

SECTION 1

Exercises 1 and 2 answered together:

a. Not a sentence

b. Informal notation

```
~Q↔~R
    F
   /\
~Q    ~R
 F     T
 |     |
 Q     R
 T     F
```

c. Official notation

```
~(Q↔R)
    T
    |
  Q↔R
    F
   /\
  Q    R
  T    F
```

d. Not a sentence

e. Informal notation

```
(P→Q) ∨ (R→~Q)
        T
       /\
 P→Q       R→~Q
              T
   /\        /\
 P   Q      R  ~Q
            F   |
                Q
```

f. Not a sentence

g. Informal notation

```
P∧Q → (Q→R∨Q)
      T
      /\
 P∧Q   Q→R∨Q
  /\      T
P    Q    /\
        Q   R∨Q
              T
             /\
            R   Q
                T
```

h. Informal notation

```
P ↔ (P↔Q∧R)
   F
   /\
 P   P↔Q∧R
 T      F
        /\
     P   Q∧R
     T     F
           /\
        Q   R
            F
```

i. Informal notation

```
P ∨ (Q→P)
  T
  /\
P   Q→P
T     /\
    Q   P
```

SECTION 2

1. a. $R \wedge P$
   b. $W \vee R$
   c. $\sim R \wedge T$
   d. $R \wedge S$
   e. $Q \leftrightarrow R$

2 and 3 answered together

a.	$S \vee V$	false
b.	$R \leftrightarrow S$	true
c.	$R \wedge S$	false
d.	$Q \vee T$	false
e.	$Q \wedge S$	false

SECTION 3

1. a.

```
~(P ∨ (Q ∧ R))
      F
      |
  P ∨ (Q ∧ R)
      T
      /\
  P   Q ∧ R
  T      /\
       Q   R
```

b.

```
~P ∨ (Q ∧ R)
   F
   /\
~P   Q∧R
 F     F
 |     /\
 P   Q   R
 T   F   F
```

c.

```
~(P ∨ R) ↔ ~P ∨ R
          T
          /\
 ~(P ∨ R)    ~P ∨ R
     F          F
     |          /\
   P ∨ R      ~P   R
     T         F   F
     /\        |
   P   R       P
   T           T
```

d.

```
~Q ∧ (P ∨ (Q↔R))
     T
     /\
 ~Q   P ∨ (Q↔R)
  T       T
  |       /\
  Q    P   Q↔R
  F         T
            /\
           Q   R
           F   F
```

e.

```
P → (~Q ↔ (~R → Q))
  F
  /\
 P    ~Q ↔ (~R → Q)
 T        F
          /\
    ~Q     ~R → Q
     T        F
     |        /\
     Q      ~R   Q
     F       T   F
             |
             R
             F
```

2. a. $\sim V \leftrightarrow \sim W$; "won't" is a negation with narrow scope.
   b. $\sim V \rightarrow (Y \rightarrow W \wedge V)$; "both" gives rise to a conjunction with narrow scope since it splits the names from the predicate. The comma prevents Y and V from occurring together.
   c. $Y \vee (W \vee V)$; "unless" is a disjunction sign.
   d. $(Y \wedge \sim V) \vee (V \wedge \sim W)$

3. a. Only if Veronica doesn't leave will William leave, or Veronica and William and Yolanda will all leave.
   (Only if Veronica doesn't leave will William leave) ∨ (Veronica and William and Yolanda will leave)
   (William will leave → Veronica doesn't leave) ∨ ($V \wedge W \wedge Y$)
   $(W \rightarrow \sim V) \vee (V \wedge W \wedge Y)$

   b. If neither William nor Veronica leaves, Yolanda won't either
   If neither William [leaves] nor Veronica leaves, [then] Yolanda won't [leave]
   $\sim(W \vee V) \rightarrow \sim Y$

   c. If William will leave if Veronica leaves, then he will surely leave if Yolanda leaves
   If (William will leave if Veronica leaves) then ([William] will leave if Yolanda leaves)
   $(V \rightarrow W) \rightarrow (Y \rightarrow W)$

d. Neither William nor Veronica nor Yolanda will leave
$\sim(W \vee V \vee Y)$

4. " Veronica leaves but neither William nor Yolanda leaves" corresponds to the truth-value assignment: V --- true; W --- false; Y --- false. We use parse trees to compute the truth values of the complex sentences.

a.

```
(W → ~V) ∨ (V ∧ W ∧ Y)
            T
           /\
 W → ~V     V ∧ W ∧ Y
   T               F
   /\              /\
 W    ~V      V ∧ W    Y
 F    F         F      F
      |         /\
      V        V   W
      T        T   F
```

b.

```
~(W ∨ V) → ~Y
           T
          /\
 ~(W ∨ V)   ~Y
    F        T
    |        |
  W ∨ V      Y
    T        F
   /\
  W   V
  F   T
```

c.

```
(V → W) → (Y → W)
           T
          /\
  V → W   Y → W
    F       T
   /\      /\
  V   W   Y   W
  T   F   F   F
```

d.

```
~(W ∨ V ∨ Y)
     F
     /\
 W ∨ V ∨ Y
       T
      /\
 W ∨ V    Y
   T      F
  /\
 W    V
 F    T
```

5. a. Sally will run and win unless she quits
(Sally will run and [Sally will] win) $\vee$ ([Sally] quits)
$(R \wedge W) \vee Q$

b. Sally will win exactly in case she runs without quitting
Sally will win exactly in case (she runs [and doesn't] quit)
$W \leftrightarrow (R \wedge {\sim}Q)$

c. Sally, who will run, will win if she doesn't quit
Sally will run, and Sally will win if she doesn't quit
$R \wedge ({\sim}Q \rightarrow W)$

d. Sally will run and quit, but she will win anyway
Sally will run and quit, and she will win
$(R \wedge Q) \wedge W$

SECTION 4

1. a. None; if we had ~~Q instead of Q it would be an instance of MTP.
   b. Simplification
   c. Double Negation
   d. MTP
   e. CB
   f. None.
   g. BC
   h. Adjunction
   i. None

2. a. $\sim W \leftrightarrow \sim X$ by CB; also $\sim X \leftrightarrow \sim W$ by CB
   b. ~~W by MTP
   c. Nothing
   d. ~W by S; also ~X by S
   e. $W \rightarrow \sim X$ by BC; also $\sim X \rightarrow W$ by BC
   f. Nothing

SECTION 5 Derivations of numbered theorems not given

SECTION 6 Derivations of numbered theorems not given

SECTION 7

1. a. All fine
   b. In line 8, the sentence that can be inferred from 7 by RT39 is $W \rightarrow \sim S$.

2, 3, 4, 5: Derivations of numbered theorems not given

SECTION 8

1. a. All fine
   b. Line 4: MTP does not apply;
   Line 8: BC (biconditional to conditional) does not apply; we could use CB;
   Line 11: MP does not apply to biconditionals; you have to split the biconditional into conditionals first using BC.
   c. Line 2: the result of applying DM to pr2 is $\sim Y \wedge \sim\sim Z$ rather than $\sim Y \wedge Z$.
   Line 3: NC doesn't apply; the NC would generate line 3 if line 2 were $Y \wedge \sim Z$.
   Line 4: Line 4 is not available at line 4; it may not be cited to justify itself. The sentence could be generated by applying MT to line 3 and pr1.

2. a. $U \wedge V \rightarrow X$ <use dm>

$\sim V \rightarrow Y$

$X \vee Y \rightarrow Z$

$\therefore\ \sim Z \rightarrow \sim U$

1.	~~Show~~ $\sim Z \rightarrow \sim U$	
2.	$\sim Z$	ass cd
3.	$\sim(X \vee Y)$	pr3 2 mt
4.	$\sim X \wedge \sim Y$	3 dm
5.	$\sim X$	4 s
6.	$\sim Y$	4 s
7.	$\sim(U \wedge V)$	pr1 5 mt
8.	$\sim U \vee \sim V$	7 dm
9.	$\sim\sim V$	6 pr2 mt
10.	$\sim U$	8 9 mtp cd

b. $(X \rightarrow Y) \rightarrow Z$ <use nc>

$\sim Z$

$V \rightarrow Y$

$\therefore\ \sim V$

1.	~~Show~~ $\sim V$	
2.	$\sim(X \rightarrow Y)$	pr1 pr2 mt
3.	$X \wedge \sim Y$	2 nc
4.	$\sim Y$	3 s
5.	$\sim V$	4 pr3 mt dd

c. $P \vee Q$

$Q \rightarrow S$

$U \vee \sim S$

$P \vee S \rightarrow R$

$R \rightarrow U$

$\therefore\ U$

1.	~~Show~~ U	
2.	~~Show~~ $P \rightarrow U$	
3.	P	ass cd
4.	$P \vee S$	3 add
5.	R	4 pr4 mp
6.	U	5 pr5 mp cd
7.	~~Show~~ $Q \rightarrow U$	
8.	Q	ass cd
9.	S	8 pr2 mp
10.	$\sim\sim S$	9 dn
11.	U	10 pr3 mtp cd
12.	U	pr1 2 7 sc
13.		12 dd

SECTION 9

1. a. $\sim(P \leftrightarrow Q)$ <use nb>

$R \vee P$

$\sim Q \rightarrow R$

$\therefore$ R

1.	~~Show~~ R	
2.	$\sim R$	ass id
3.	P	2 pr2 mtp
4.	$P \leftrightarrow \sim Q$	pr1 nb
5.	$P \rightarrow \sim Q$	4 bc
6.	$\sim Q$	3 5 mp
7.	R	6 pr3 mp
8.		2 7 id

b. $W \rightarrow U$ <use cdj>

$\sim W \rightarrow V$

$\therefore$ $U \vee V$

1.	~~Show~~ $U \vee V$	
2.	~~Show~~ $\sim U \rightarrow V$	
3.	$\sim U$	ass cd
4.	$\sim W$	3 pr1 mt
5.	V	4 pr2 mp cd
6.	$U \vee V$	2 cdj dd

c. $P \vee (Q \wedge S)$

$R \vee Q$

$S \vee \sim P$

$Q \rightarrow \sim S$

$\therefore$ R

1.	~~Show~~ R	
2.	$\sim R$	ass id
3.	Q	2 pr2 mtp
4.	$\sim S$	3 pr4 mp
5.	$\sim Q \vee \sim S$	4 add
6.	$\sim(Q \wedge S)$	5 dm
7.	P	6 pr1 mtp
8.	$\sim\sim P$	7 dn
9.	S	8 pr3 mtp
10.		4 9 id

SECTION 10

1. a. $(R \leftrightarrow S) \vee (R \leftrightarrow {\sim}S)$; tautology

R	S	$(R \leftrightarrow S) \vee (R \leftrightarrow {\sim}S)$
T	T	T
T	F	T
F	T	T
F	F	T

b. $R \leftrightarrow (S \leftrightarrow R)$; not a tautology

R	S	$R \leftrightarrow (S \leftrightarrow R)$
T	F	F

c. $R \vee (S \wedge T) \rightarrow R \wedge (S \vee T)$; not a tautology

R	S	T	$R \vee (S \wedge T) \rightarrow R \wedge (S \vee T)$
F	T	T	F

d. ${\sim}U \rightarrow (U \rightarrow {\sim}V)$; tautology

U	V	${\sim}U \rightarrow (U \rightarrow {\sim}V)$
T	T	T
T	F	T
F	T	T
F	F	T

e. $({\sim}R \leftrightarrow R) \rightarrow S$; tautology

R	S	$({\sim}R \leftrightarrow R) \rightarrow S$
T	T	T
T	F	T
F	T	T
F	F	T

f. $(S \wedge T) \vee (S \wedge {\sim}T) \vee {\sim}S$; tautology

T	S	$(S \wedge T) \vee (S \wedge {\sim}T) \vee {\sim}S$
T	T	T
T	F	T
F	T	T
F	F	T

SECTION 11

a. $U \wedge V \rightarrow X$ NO
 ${\sim}V \rightarrow U$
 $X \vee V \rightarrow U$
$\therefore$ $V \rightarrow {\sim}U$

U	V	X	$U \wedge V \rightarrow X$	${\sim}V \rightarrow U$	$X \vee V \rightarrow U$	$V \rightarrow {\sim}U$
T	T	T	T	T	T	F

b. $(X \rightarrow Y) \rightarrow Z$ YES
$\sim Z$
$\therefore \sim Y$

X	Y	Z	$(X \rightarrow Y) \rightarrow Z$	$\sim Z$	$\sim Y$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	T	F	T
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	F	T	F
F	F	T	T	F	T
F	F	F	F	T	T

c. $\sim(P \leftrightarrow Q)$ YES
$R \vee P$
$\sim Q \rightarrow R$
$\therefore R$

P	Q	R	$\sim(P \leftrightarrow Q)$	$R \vee P$	$\sim Q \rightarrow R$	R
T	T	T	F	T	T	T
T	T	F	F	T	T	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	F	T	T	T
F	F	F	F	F	F	F

d. $S \vee T$ NO
$W \vee S$
$\sim T \vee \sim S$
$\therefore \sim S$

S	T	W	$S \vee T$	$W \vee S$	$\sim T \vee \sim S$	$\sim S$
T	F	T	T	T	T	F

e. $W \rightarrow U$ YES
$\sim W \rightarrow V$
$\therefore U \vee V$

U	V	W	$W \rightarrow U$	$\sim W \rightarrow V$	$U \vee V$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	T
F	T	F	T	T	T
F	F	T	F	T	F
F	F	F	T	F	F

f. $P \leftrightarrow \sim Q$ NO
$Q \rightarrow R \vee P$
$R \rightarrow \sim Q \vee \sim P$
$\therefore Q \vee R$

P	Q	R	$P \leftrightarrow \sim Q$	$Q \rightarrow R \vee P$	$R \rightarrow \sim Q \vee \sim P$	$Q \vee R$
T	F	F	T	T	T	F

g. $P \vee (Q \wedge S)$ YES
$S \vee Q$
$S \vee \sim P$
$\therefore$ S

P	Q	S	$P \vee (Q \wedge S)$	$S \vee Q$	$S \vee \sim P$	S
T	T	T	T	T	T	T
T	T	F	T	T	F	F
T	F	T	T	T	T	T
T	F	F	T	F	F	F
F	T	T	T	T	T	T
F	T	F	F	T	T	F
F	F	T	F	T	T	T
F	F	F	F	F	T	F

h. $P \wedge (Q \vee S)$ NO
$S \vee Q$
$S \vee P$
$\therefore$ S

P	Q	S	$P \wedge (Q \vee S)$	$S \vee Q$	$S \vee P$	S
T	T	F	T	T	T	F