

Answers to the Exercises -- Chapter 2

SECTION 1

Exercises 1 and 2 answered together:

- a. Not a sentence
- b. Informal notation

$$\begin{array}{c} \sim Q \leftrightarrow \sim R \\ \text{F} \\ \wedge \\ \sim Q \quad \sim R \\ \text{F} \quad \text{T} \\ | \quad | \\ Q \quad R \\ \text{T} \quad \text{F} \end{array}$$

- c. Official notation

$$\begin{array}{c} \sim(Q \leftrightarrow R) \\ \text{T} \\ | \\ Q \leftrightarrow R \\ \text{F} \\ \wedge \\ Q \quad R \\ \text{T} \quad \text{F} \end{array}$$

- d. Not a sentence
- e. Informal notation

$$\begin{array}{c} (P \rightarrow Q) \vee (R \rightarrow \sim Q) \\ \text{T} \\ \wedge \\ P \rightarrow Q \quad R \rightarrow \sim Q \\ \wedge \quad \wedge \\ P \quad Q \quad R \quad \sim Q \\ \quad \quad \quad \text{F} \quad | \\ \quad \quad \quad \quad \quad Q \end{array}$$

- f. Not a sentence
- g. Informal notation

$$\begin{array}{c} P \wedge Q \rightarrow (Q \rightarrow R \vee Q) \\ \text{T} \\ \wedge \\ P \wedge Q \quad Q \rightarrow R \vee Q \\ \wedge \quad \wedge \\ P \quad Q \quad Q \quad R \vee Q \\ \quad \quad \quad \wedge \\ \quad \quad \quad R \quad Q \\ \quad \quad \quad \text{T} \end{array}$$

h. Informal notation

$$\begin{array}{c}
 P \leftrightarrow (P \leftrightarrow Q \wedge R) \\
 \text{F} \\
 \wedge \\
 P \quad P \leftrightarrow Q \wedge R \\
 \text{T} \quad \text{F} \\
 \wedge \\
 P \quad Q \wedge R \\
 \text{T} \quad \text{F} \\
 \wedge \\
 Q \quad R \\
 \text{F}
 \end{array}$$

i. Informal notation

$$\begin{array}{c}
 P \vee (Q \rightarrow P) \\
 \text{T} \\
 \wedge \\
 P \quad Q \rightarrow P \\
 \text{T} \quad \wedge \\
 Q \quad P
 \end{array}$$

SECTION 2

1. a. $R \wedge P$
- b. $W \vee R$
- c. $\sim R \wedge T$
- d. $R \wedge S$
- e. $Q \leftrightarrow R$

2 and 3 answered together

- a. $S \vee V$ false
- b. $R \leftrightarrow S$ true
- c. $R \wedge S$ false
- d. $Q \vee T$ false
- e. $Q \wedge S$ false

SECTION 3

1. a.

$$\begin{array}{c}
 \sim(P \vee (Q \wedge R)) \\
 \text{F} \\
 | \\
 P \vee (Q \wedge R) \\
 \text{T} \\
 \wedge \\
 P \quad Q \wedge R \\
 \text{T} \quad \wedge \\
 Q \quad R
 \end{array}$$

b.

$$\begin{array}{c}
 \sim P \vee (Q \wedge R) \\
 \text{F} \\
 \wedge \\
 \sim P \quad Q \wedge R \\
 \text{F} \quad \text{F} \\
 | \quad \wedge \\
 P \quad Q \quad R \\
 \text{T} \quad \text{F} \quad \text{F}
 \end{array}$$

c. $\sim(P \vee R) \leftrightarrow \sim P \vee R$

$$\begin{array}{c} \text{T} \\ \wedge \\ \sim(P \vee R) \quad \sim P \vee R \\ \text{F} \quad \text{F} \\ | \quad \wedge \\ P \vee R \quad \sim P \quad R \\ \text{T} \quad \text{F} \quad \text{F} \\ \wedge \\ P \quad R \\ \text{T} \quad \text{T} \end{array}$$

d. $\sim Q \wedge (P \vee (Q \leftrightarrow R))$

$$\begin{array}{c} \text{T} \\ \wedge \\ \sim Q \quad P \vee (Q \leftrightarrow R) \\ \text{T} \quad \text{T} \\ | \quad \wedge \\ Q \quad P \quad Q \leftrightarrow R \\ \text{F} \quad \text{T} \quad \text{T} \\ \wedge \\ Q \quad R \\ \text{F} \quad \text{F} \end{array}$$

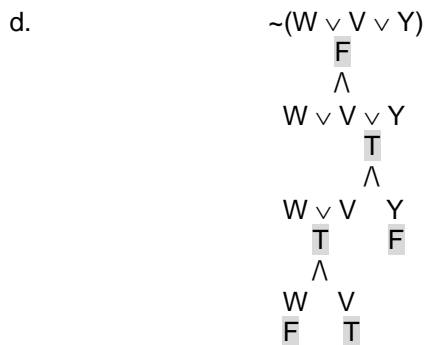
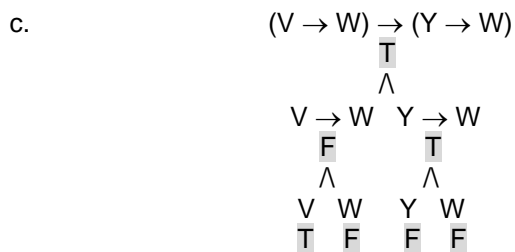
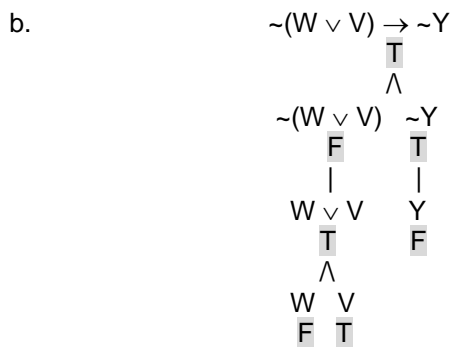
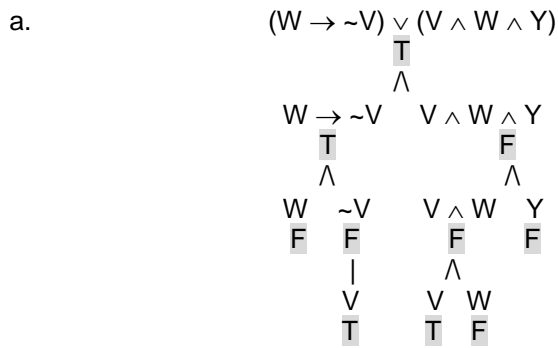
e. $P \rightarrow (\sim Q \leftrightarrow (\sim R \rightarrow Q))$

$$\begin{array}{c} \text{F} \\ \wedge \\ P \quad \sim Q \leftrightarrow (\sim R \rightarrow Q) \\ \text{T} \quad \text{F} \\ \wedge \\ \sim Q \quad \sim R \rightarrow Q \\ \text{T} \quad \text{F} \\ | \quad \wedge \\ Q \quad \sim R \quad Q \\ \text{F} \quad \text{T} \quad \text{F} \\ | \\ R \\ \text{F} \end{array}$$

2. a. $\sim V \leftrightarrow \sim W$; "won't" is a negation with narrow scope.
 b. $\sim V \rightarrow (Y \rightarrow W \wedge V)$; "both" gives rise to a conjunction with narrow scope since it splits the names from the predicate. The comma prevents Y and V from occurring together.
 c. $Y \vee (W \vee V)$; "unless" is a disjunction sign.
 d. $(Y \wedge \sim V) \vee (V \wedge \sim W)$
3. a. Only if Veronica doesn't leave will William leave, or Veronica and William and Yolanda will all leave.
 (Only if Veronica doesn't leave will William leave) \vee (Veronica and William and Yolanda will leave)
 $(W \rightarrow \sim V) \vee (V \wedge W \wedge Y)$
- b. If neither William nor Veronica leaves, Yolanda won't either
 If neither William [leaves] nor Veronica leaves, [then] Yolanda won't [leave]
 $\sim(W \vee V) \rightarrow \sim Y$
- c. If William will leave if Veronica leaves, then he will surely leave if Yolanda leaves
 If (William will leave if Veronica leaves) then ([William] will leave if Yolanda leaves)
 $(V \rightarrow W) \rightarrow (Y \rightarrow W)$

- d. Neither William nor Veronica nor Yolanda will leave
 $\sim(W \vee V \vee Y)$

4. "Veronica leaves but neither William nor Yolanda leaves" corresponds to the truth-value assignment: V --- true; W --- false; Y --- false. We use parse trees to compute the truth values of the complex sentences.



5. a. Sally will run and win unless she quits
 (Sally will run and [Sally will] win) \vee ([Sally] quits)
 $(R \wedge W) \vee Q$
- b. Sally will win exactly in case she runs without quitting
 Sally will win exactly in case (she runs [and doesn't] quit)
 $W \leftrightarrow (R \wedge \sim Q)$
- c. Sally, who will run, will win if she doesn't quit
 Sally will run, and Sally will win if she doesn't quit
 $R \wedge (\sim Q \rightarrow W)$
- d. Sally will run and quit, but she will win anyway
 Sally will run and quit, and she will win
 $(R \wedge Q) \wedge W$

SECTION 4

1. a. None; if we had $\sim\sim Q$ instead of Q it would be an instance of MTP.
 b. Simplification
 c. Double Negation
 d. MTP
 e. CB
 f. None.
 g. BC
 h. Adjunction
 i. None
2. a. $\sim W \leftrightarrow \sim X$ by CB; also $\sim X \leftrightarrow \sim W$ by CB
 b. $\sim\sim W$ by MTP
 c. Nothing
 d. $\sim W$ by S; also $\sim X$ by S
 e. $W \rightarrow \sim X$ by BC; also $\sim X \rightarrow W$ by BC
 f. Nothing

SECTION 5 Derivations of numbered theorems not given

SECTION 6 Derivations of numbered theorems not given

SECTION 7

1. a. All fine
 b. In line 8, the sentence that can be inferred from 7 by RT39 is $W \rightarrow \sim S$.
- 2, 3, 4, 5: Derivations of numbered theorems not given

SECTION 8

1. a. All fine
 b. Line 4: MTP does not apply;
 Line 8: BC (biconditional to conditional) does not apply; we could use CB;
 Line 11: MP does not apply to biconditionals; you have to split the biconditional into conditionals first using BC.
- c. Line 2: the result of applying DM to pr2 is $\sim Y \wedge \sim\sim Z$ rather than $\sim Y \wedge Z$.
 Line 3: NC doesn't apply; the NC would generate line 3 if line 2 were $Y \wedge \sim Z$.
 Line 4: Line 4 is not available at line 4; it may not be cited to justify itself. The sentence could be generated by applying MT to line 3 and pr1.

2. a. $U \wedge V \rightarrow X$ <use dm>
 $\sim V \rightarrow Y$
 $X \vee Y \rightarrow Z$
 $\therefore \sim Z \rightarrow \sim U$

1. Show $\sim Z \rightarrow \sim U$

2.	$\sim Z$	ass cd
3.	$\sim(X \vee Y)$	pr3 2 mt
4.	$\sim X \wedge \sim Y$	3 dm
5.	$\sim X$	4 s
6.	$\sim Y$	4 s
7.	$\sim(U \wedge V)$	pr1 5 mt
8.	$\sim U \vee \sim V$	7 dm
9.	$\sim \sim V$	6 pr2 mt
10.	$\sim U$	8 9 mtp cd

- b. $(X \rightarrow Y) \rightarrow Z$ <use nc>
 $\sim Z$
 $V \rightarrow Y$
 $\therefore \sim V$

1. Show $\sim V$

2.	$\sim(X \rightarrow Y)$	pr1 pr2 mt
3.	$X \wedge \sim Y$	2 nc
4.	$\sim Y$	3 s
5.	$\sim V$	4 pr3 mt dd

- c. $P \vee Q$
 $Q \rightarrow S$
 $U \vee \sim S$
 $P \vee S \rightarrow R$
 $R \rightarrow U$
 $\therefore U$

1. Show U

2.	Show $P \rightarrow U$	
3.	P	ass cd
4.	$P \vee S$	3 add
5.	R	4 pr4 mp
6.	U	5 pr5 mp cd
7.	Show $Q \rightarrow U$	
8.	Q	ass cd
9.	S	8 pr2 mp
10.	$\sim \sim S$	9 dn
11.	U	10 pr3 mtp cd
12.	U	pr1 2 7 sc
13.		12 dd

SECTION 9

1. a. $\sim(P \leftrightarrow Q)$ <use nb>
 $R \vee P$
 $\sim Q \rightarrow R$
 $\therefore R$

1. Show R

2.	$\sim R$	ass id
3.	P	2 pr2 mtp
4.	$P \leftrightarrow \sim Q$	pr1 nb
5.	$P \rightarrow \sim Q$	4 bc
6.	$\sim Q$	3 5 mp
7.	R	6 pr3 mp
8.		2 7 id

- b. $W \rightarrow U$ <use cdj>
 $\sim W \rightarrow V$
 $\therefore U \vee V$

1. Show $U \vee V$

2.	Show $\sim U \rightarrow V$	
3.	$\sim U$	ass cd
4.	$\sim W$	3 pr1 mt
5.	V	4 pr2 mp cd
6.	$U \vee V$	2 cdj dd

- c. $P \vee (Q \wedge S)$
 $R \vee Q$
 $S \vee \sim P$
 $Q \rightarrow \sim S$
 $\therefore R$

1. Show R

2.	$\sim R$	ass id
3.	Q	2 pr2 mtp
4.	$\sim S$	3 pr4 mp
5.	$\sim Q \vee \sim S$	4 add
6.	$\sim(Q \wedge S)$	5 dm
7.	P	6 pr1 mtp
8.	$\sim \sim P$	7 dn
9.	S	8 pr3 mtp
10.		4 9 id

SECTION 10

1. a. $(R \leftrightarrow S) \vee (R \leftrightarrow \neg S)$; tautology

R	S	$(R \leftrightarrow S) \vee (R \leftrightarrow \neg S)$
T	T	T
T	F	T
F	T	T
F	F	T

b. $R \leftrightarrow (S \leftrightarrow R)$; not a tautology

R	S	$R \leftrightarrow (S \leftrightarrow R)$
T	F	F

c. $R \vee (S \wedge T) \rightarrow R \wedge (S \vee T)$; not a tautology

R	S	T	$R \vee (S \wedge T) \rightarrow R \wedge (S \vee T)$
F	T	T	F

d. $\neg U \rightarrow (U \rightarrow \neg V)$; tautology

U	V	$\neg U \rightarrow (U \rightarrow \neg V)$
T	T	T
T	F	T
F	T	T
F	F	T

e. $(\neg R \leftrightarrow R) \rightarrow S$; tautology

R	S	$(\neg R \leftrightarrow R) \rightarrow S$
T	T	T
T	F	T
F	T	T
F	F	T

f. $(S \wedge T) \vee (S \wedge \neg T) \vee \neg S$; tautology

T	S	$(S \wedge T) \vee (S \wedge \neg T) \vee \neg S$
T	T	T
T	F	T
F	T	T
F	F	T

SECTION 11

a. $U \wedge V \rightarrow X$ NO
 $\neg V \rightarrow U$
 $X \vee V \rightarrow U$
 $\therefore V \rightarrow \neg U$

U	V	X	$U \wedge V \rightarrow X$	$\neg V \rightarrow U$	$X \vee V \rightarrow U$	$V \rightarrow \neg U$
T	T	T	T	T	T	F

b. $(X \rightarrow Y) \rightarrow Z$ YES

$\sim Z$

$\therefore \sim Y$

X	Y	Z	$(X \rightarrow Y) \rightarrow Z$	$\sim Z$	$\sim Y$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	T	F	T
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	F	T	F
F	F	T	T	F	T
F	F	F	F	T	T

c. $\sim(P \leftrightarrow Q)$ YES

$R \vee P$

$\sim Q \rightarrow R$

$\therefore R$

P	Q	R	$\sim(P \leftrightarrow Q)$	$R \vee P$	$\sim Q \rightarrow R$	R
T	T	T	F	T	T	T
T	T	F	F	T	T	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	F	T	T	T
F	F	F	F	F	F	F

d. $S \vee T$ NO

$W \vee S$

$\sim T \vee \sim S$

$\therefore \sim S$

S	T	W	$S \vee T$	$W \vee S$	$\sim T \vee \sim S$	$\sim S$
T	F	T	T	T	T	F

e. $W \rightarrow U$ YES

$\sim W \rightarrow V$

$\therefore U \vee V$

U	V	W	$W \rightarrow U$	$\sim W \rightarrow V$	$U \vee V$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	T
F	T	F	T	T	T
F	F	T	F	T	F
F	F	F	T	F	F

f. $P \leftrightarrow \sim Q$ NO

$Q \rightarrow R \vee P$

$R \rightarrow \sim Q \vee \sim P$

$\therefore Q \vee R$

P	Q	R	$P \leftrightarrow \sim Q$	$Q \rightarrow R \vee P$	$R \rightarrow \sim Q \vee \sim P$	$Q \vee R$
T	F	F	T	T	T	F

g. $P \vee (Q \wedge S)$ YES
 $S \vee Q$
 $S \vee \sim P$
 $\therefore S$

P	Q	S	$P \vee (Q \wedge S)$	$S \vee Q$	$S \vee \sim P$	S
T	T	T	T	T	T	T
T	T	F	T	T	F	F
T	F	T	T	T	T	T
T	F	F	T	F	F	F
F	T	T	T	T	T	T
F	T	F	F	T	T	F
F	F	T	F	T	T	T
F	F	F	F	F	T	F

h. $P \wedge (Q \vee S)$ NO
 $S \vee Q$
 $S \vee P$
 $\therefore S$

P	Q	S	$P \wedge (Q \vee S)$	$S \vee Q$	$S \vee P$	S
T	T	F	T	T	T	F