Answers to the Exercises -- Chapter 3

SECTION 1

1. a. Fred is an orangutan.
   Of

   b. Gertrude is an orangutan but Fred isn't.
   Gertrude is an orangutan [and] Fred is not [an orangutan].
   Og ∧ ~Of

   c. Tony Blair will speak first.
   Fb

   d. Gary lost weight recently; he is happy.
   Gary lost weight recently [and] [Gary] is happy.
   Lg ∧ Hg

   e. Felix cleaned and polished.
   Felix cleaned and [Felix] polished.
   Cf ∧ Of

   f. Darlene or Abe will bat clean-up.
   Darlene [will bat clean-up] or Abe will bat clean-up.
   Bd ∨ Ba

2. 'D' is true of doctors
   'L' is true of people who are in love
   'h' stands for Hans
   'a' stands for Amanda

   a. *Hans is a doctor but Amanda isn't.*
   Hans is a doctor [and] Amanda is not [a doctor]
   Dh ∧ ~Da

   b. *Hans, who is a doctor, is in love*
   Hans is in love [and Hans] is a doctor
   Lh ∧ Dh

   c. *Hans is in love but Amanda isn't*
   Hans is in love [and] Amanda is [not in love]
   Lh ∧ ~La

   d. *Neither Hans nor Amanda is in love*
   [It is not the case that] (Hans [is in love] or Amanda is in love)
   ~(Lh ∨ La)

   f. *Hans and Amanda are both doctors*.
   Hans is a doctor [and] Amanda is a doctor.
   Dh ∧ Da

3. 'L' for things that live in Brea
   'D' for things that drive to school

   a. *Eileen and Cosi both live in Brea*.
   Eileen lives in Brea and Cosi loves in Brea
   Le ∧ Lc

   b. *Eileen drives to school, and so does Hank*.
   Eileen drives to school and hank drives to school
   De ∧ Dh

c. *If Hank lives in Brea then he drives to school; otherwise he doesn't drive to school.*
(If Hank lives in Brea then he drives to school) [and] (otherwise he doesn't drive to school)
(If Hank lives in Brea then he drives to school) [and] ([if Hank doesn't live in Brea then] he doesn't drive to school)
$(Lh \to Dh) \land (\sim Lh \to \sim Dh)$

d. *If David and Hank both live in Brea then David drives to school but Hank doesn't.*
If (David and Hank both live in Brea) then (David drives to school [and] Hank doesn't [drive to school])
$(Ld \land Lh) \to (Dd \land \sim Dh)$

e. *Neither Hank nor Eileen live in Brea, yet each of them drives to school.*
Neither Hank nor Eileen live in Brea, [and] [Hank and Eileen] drive to school.
$\sim(Lh \lor Le) \land (Dh \land De)$

SECTION 2

1. For each of the following, say whether it is a formula in official notation, or in informal notation, or not a formula at all. If it is a formula, parse it.

a. Official notation

```
  ~∀x(Fx → (Gx ∧ Hx))
             |
    ∀x(Fx → (Gx ∧ Hx))
     |
(Fx → (Gx ∧ Hx))
    /\
 Fx   (Gx ∧ Hx)
         /\
       Gx   Hx
```

b. Informal notation

```
∃x~~Gx → Hx ∨ ∃yGy
         /\
 ∃x~~Gx   Hx ∨ ∃yGy
  |            /\
~~Gx       Hx   ∃yGy
   |              |
 ~Gx             Gy
   |
  Gx
```

c. Official notation

```
~(Gx ↔ ~Hx)
      |
 (Gx ↔ ~Hx)
     /\
  Gx   ~Hx
         |
        Hx
```

d. Not a formula; a quantifier cannot occur outside a quantifier phrase.

e. Informal notation

```
Fa → (Gb ↔ Hc)
    /\
 Fa   (Gb ↔ Hc)
          /\
        Gb   Hc
```

f. Not a formula; a variable can only occur in an atomic formula or a quantifier phrase, and never by itself.

g. Informal notation

```
      ∀x(Gx ↔ Hx) → Ha ∧ ∃zKz
                   /\
       ∀x(Gx ↔ Hx)   Ha ∧ ∃zKz
        |                /\
   Gx ↔ Hx           Ha   ∃zKz
       /\                  |
    Gx   Hx                Kz
```

SECTION 3

1. a. Sentence $\exists x(Fx \wedge \forall y(Gy \vee Hx))$

   b. Not a formula; there is no way to form "∃~z" in our grammar.

   c. Formula $\exists z{\sim}(Hz \wedge Gx \wedge \exists xIx)$

   d. Formula ${\sim}({\sim}Gx \rightarrow \forall y(Jx \wedge Ky \leftrightarrow Lx))$

   e. Formula $\exists xGx \leftrightarrow \exists y(Gy \wedge Hx)$

   f. Sentence $\forall x(Gx \rightarrow \forall y(Hy \rightarrow \forall z(Iz \rightarrow Hx \wedge Gz)))$

   g. Sentence $\forall x \exists y(Hx \leftrightarrow {\sim}Gy)$

   h. Not a formula; there is no way to form "∀xy" in our grammar.

   i. Not a formula; "∃y" cannot stand on its own as a subformula.

   j. Sentence $\forall x \exists y \forall z(Gx \leftrightarrow \exists w(Hw \wedge {\sim}Hx \wedge Gy))$

SECTION 4

1. a. Something is a sofa and is well built. There is a well-built sofa..
   b. Everything is such that if it is a sofa then it is well-built. All sofas are well-built.
   c. Everything is either a sofa or is well-built. Everything is a sofa, unless it's well-built.
   d. Something is such that it is not a sofa. Something isn't a sofa.
   e. Everything is such that it is not a sofa. There are no sofas.
   f. Everything is such that if it is both bell-built and a sofa, then it is comfortable. Every well-built sofa is comfortable.
   g. Something is comfortable and everything is well-built.
   h. Something is such that if it is comfortable, then everything is well-built.

2. Assume that all giraffes are friendly, and that some giraffes are clever and some aren't.

   a. $\forall x(Gx \rightarrow Fx)$ True, since all giraffes are friendly.
   b. $\forall x(Gx \rightarrow Cx)$ False, since not every giraffe is clever.
   c. $\exists x({\sim}Fx \wedge Gx)$ False, since every giraffe is friendly.
   d. $\exists y(Fy \wedge Cy)$ True, since giraffes are friendly, and some of them are clever.
   e. $\exists z(Gz \wedge Cz)$ True, since some giraffes are clever.
   f. $\forall x(Gx \rightarrow {\sim}Gx)$ False, since not every giraffe isn't a giraffe. (In fact, no giraffe isn't a giraffe, but it only takes one to falsify the symbolic sentence.)

SECTION 5a

1. a. Every **H**andsome **E**lephant is **F**riendly.
   $\forall x((Hx \wedge Ex) \rightarrow Fx)$
   b. No handsome elephant is friendly.
   $\sim\exists x((Hx \wedge Ex) \wedge Fx)$
   c. Some elephants are not handsome.
   $\exists x(Ex \wedge \sim Hx)$
   d. Some handsome elephants are friendly.
   $\exists x((Hx \wedge Ex) \wedge Fx)$
   e. Each friendly elephant is handsome.
   $\forall x((Fx \wedge Ex) \rightarrow Hx)$
   f. A handsome elephant is not friendly.
   $\exists x((Hx \wedge Ex) \wedge \sim Fx)$
   g. No friendly elephant is handsome.
   $\sim\exists x((Fx \wedge Ex) \wedge Hx)$

SECTION 5b

1. Suppose that \`A' stands for \`is a U.S. state', \`C' for \`is a city', \`L' for \`is a capital', and \`E' for \`is in the Eastern time zone'. What are the truth values of these sentences?

   a. $\forall x(Cx \rightarrow Lx)$ --- False; Los Angeles is a city but not a capital.
   b. $\exists x(Cx \wedge Lx)$ --- True; Sacramento is a city and a capital.
   c. $\exists x(Cx \wedge Lx \leftrightarrow Ex)$ --- True, because something makes the biconditional true, by making both sides false. For example, Los Angeles is not a capital, and it is not in the Eastern time zone.
   d. $\forall x(Cx \wedge Ex \rightarrow Ax)$ --- False; Philadelphia is not a state.
   e. $\sim\exists x(Ax \wedge Ex)$ --- False; Delaware is a state in the Eastern time zone.
   f. $\exists x(Cx \wedge Ex) \wedge \exists x(Cx \wedge \sim Ex)$ --- True; Philadelphia is a city in the Eastern time zone and LA is a city outside the eastern time zone.
   g. $\exists x(Cx \wedge Ex \wedge Ax)$ --- False; no city is also a state.
   h. $\sim\exists x(Cx \wedge \sim Cx)$ --- True. There is no city which isn't a city.

2. a. All **G**iraffes are sp**O**tted.
   $\forall x(Gx \rightarrow Ox)$
   b. All **C**lever giraffes are spotted.
   $\forall x(Gx \wedge Cx \rightarrow Ox)$
   c. No clever giraffes are spotted.
   $\sim\exists x(Gx \wedge Cx \wedge Ox)$
   d. Every giraffe is either spotted or **D**rab.
   $\forall x(Gx \rightarrow (Ox \vee Dx))$
   e. Some giraffes are clever.
   $\exists x(Gx \wedge Cx)$
   f. Some spotted giraffes are clever.
   $\exists x(Ox \wedge Gx \wedge Cx)$
   g. Some giraffes are clever and some aren't.
   Some giraffes are clever and some [giraffes are not clever].
   $\exists x(Gx \wedge Cx) \wedge \exists x(Gx \wedge \sim Cx)$
   h. Some spotted giraffes aren't clever.
   $\exists x(Ox \wedge Gx \wedge \sim Cx)$
   i. No spotted giraffe is clever but every unspotted one is.
   No spotted giraffe is clever [and] every un-spotted [giraffe] is [clever].
   $\sim\exists x(Ox \wedge Gx \wedge Cx) \wedge \forall x(\sim Ox \wedge Gx \rightarrow Cx)$
   j. Every clever spotted giraffe is either w**I**se or **F**oolhardy.
   $\forall x(((Cx \wedge Sx) \wedge Gx) \rightarrow (Ix \vee Fx))$
   k. Either all spotted giraffes are clever, or all clever giraffes are spotted.

$\forall x(Ox \wedge Gx \rightarrow Cx) \vee \forall x($

l. Every clever giraffe is foolhardy.
$\forall x(Cx \wedge Gx \rightarrow Fx)$

m. If some giraffes are wise then not all giraffes are foolhardy.
$\exists x(Gx \wedge Ix) \rightarrow {\sim}\forall x(Gx \rightarrow Fx)$

n. All giraffes are spotted if and only if no giraffes aren't spotted.
$\forall x(Gx \rightarrow Ox) \leftrightarrow {\sim}\exists x(Gx \wedge {\sim}Ox)$

o. Nothing is both wise and foolhardy.
${\sim}\exists x(Ix \wedge Fx)$

SECTION 5c

1. a. Only **F**riendly **E**lephants are **H**andsome (ambiguous)
   i. $\forall x(Hx \rightarrow (Fx \wedge Ex))$
   ii. $\forall x((Ex \wedge Hx) \rightarrow Fx)$

b. If only elephants are friendly, no **G**iraffes are friendly
$\forall x(Fx \rightarrow Ex) \rightarrow {\sim}\exists x(Gx \wedge Fx)$

c. Only the **B**rave are f**A**ir.
$\forall x(Ax \rightarrow Bx)$

d. If only elephants are friendly then every elephant is friendly
$\forall x(Fx \rightarrow Ex) \rightarrow \forall x(Ex \rightarrow Fx)$

e. All and only elephants are friendly.
All elephants are friendly [and] Only elephants are friendly.
$\forall x(Ex \rightarrow Fx) \wedge \forall x(Fx \rightarrow Ex)$

f. If every elephant is friendly, only friendly **A**nimals are elephants (ambiguous)
i. $\forall x(Ex \rightarrow Fx) \rightarrow \forall x(Ex \rightarrow (Fx \wedge Ax))$
ii. $\forall x(Ex \rightarrow Fx) \rightarrow \forall x((Ex \wedge Ax) \rightarrow Fx)$

g. If any elephants are friendly, all and only giraffes are nasty
If some elephants are friendly, (all giraffes are **N**asty and only giraffes are nasty)
$\exists x(Ex \wedge Fx) \rightarrow (\forall x(Gx \rightarrow Nx) \wedge \forall x(Nx \rightarrow Gx))$

h. Among sp**O**tted animals, only giraffes are handsome.
$\forall x(Ox \rightarrow (Hx \rightarrow Gx))$

i. Among spotted animals, all and only giraffes are handsome
$\forall x(Ox \rightarrow ((Gx \rightarrow Hx) \wedge (Hx \rightarrow Gx))$

j. Only giraffes frolic if annoyed.
If a thing fro**L**ics if a**N**noyed, it is a giraffe.
$\forall x((Nx \rightarrow Lx) \rightarrow Gx)$

SECTION 5d

1. Symbolize these sentences.

a. Every **G**iraffe which **F**rolics is **H**appy
$\forall x(Fx \wedge Gx \rightarrow Hx)$

b. Only giraffes which frolic are happy (ambiguous)
i. $\forall x(Gx \wedge Hx \rightarrow Fx)$
ii. $\forall x(Hx \rightarrow Gx \wedge Fx)$

c. Only giraffes are **A**nimals which are **L**ong-necked.
$\forall x(Ax \wedge Lx \rightarrow Gx)$

d. If only giraffes frolic, every animal which is not a giraffe doesn't frolic.
$\forall x(Fx \rightarrow Gx) \rightarrow \forall x(Ax \wedge {\sim}Gx \rightarrow {\sim}Fx)$

e. Some giraffe which frolics is long-necked or happy.
$\exists x((Fx \wedge Gx) \wedge (Lx \vee Hx))$

f. No giraffe which is not happy frolics and is long-necked.
${\sim}\exists x(({\sim}Hx \wedge Gx) \wedge (Fx \wedge Lx))$

g. Some giraffe is not both long-necked and happy.
$\exists x(Gx \wedge {\sim}(Lx \wedge Hx))$

SECTION 5e

1. a. If a **G**iraffe is **H**appy then it **F**rolics unless it is **L**ame.
   $\forall x(Gx \wedge Hx \rightarrow Fx \vee Lx)$
   b. A **M**onkey frolics unless it is not happy.
   $\forall x(Mx \rightarrow Fx \vee \sim Hx)$
   c. Among giraffes, only happy ones frolic.
   $\forall x(Gx \rightarrow (Fx \rightarrow Hx))$
   d. All and only giraffes are happy if they are not lame.
   $\forall x(Gx \leftrightarrow (\sim Lx \rightarrow Hx))$
   e. A giraffe frolics only if it is happy.
   $\forall x(Gx \wedge Fx \rightarrow Hx)$ or $\forall x(Gx \rightarrow (Fx \rightarrow Hx))$
   f. Only giraffes frolic if happy.
   $\forall x((Hx \rightarrow Fx) \rightarrow Gx)$
   g. All monkeys are happy if some giraffe is.
   $\exists x(Gx \wedge Hx) \rightarrow \forall x(Mx \rightarrow Hx)$
   h. **C**ute monkeys frolic.
   $\forall x(Cx \wedge Mx \rightarrow Fx)$
   i. Giraffes ru**N** and frolic if and only if they are **B**lissful and **E**xultant.
   $\forall x(Gx \rightarrow (Ux \wedge Fx \leftrightarrow Bx \wedge Ex))$
   j. If those who are he**A**lthy are not lame, then if they are exultant, they will frolic.
   $\forall x((Ax \rightarrow \sim Lx) \rightarrow (Ex \rightarrow Fx))$
   k. Only giraffes and monkeys are blissful and exultant.
   $\forall x(Bx \wedge Ex \rightarrow Gx \vee Mx)$
   l. The brave(**I**) are happy.
   $\forall x(Ix \rightarrow Hx)$
   m. If a giraffe frolics, then no monkey is blissful unless it is.
   $\forall x((Gx \wedge Fx) \rightarrow (Bx \vee \sim\exists y(My \wedge By)))$
   n. Giraffes and monkeys frolic if happy.
   $\forall x(Gx \vee Mx \rightarrow (Hx \rightarrow Fx))$

SECTION 6

1. a. Th**e** sky is **B**lue
   Everything that is blue is pr**E**tty
   ∴ Something is pretty

   Be
   $\forall x(Bx \rightarrow Ex)$
   ∴ $\exists xEx$

1	~~Show~~ $\exists xEx$	
2	$Be \rightarrow Ee$	pr2 ui
3	Ee	2 pr1 mp
4	$\exists xEx$	3 eg dd

b. Every **H**yena is **G**rey.
Every hyena is an **A**nimal
J**e**nny is a hyena
∴ Some animal is grey

$\forall x(Hx \rightarrow Gx)$
$\forall x(Hx \rightarrow Ax)$
He
∴ $\exists x(Ax \wedge Gx)$

1	~~Show~~ $\exists x(Ax \wedge Gx)$	
2	$He \rightarrow Ge$	pr1 ui
3	$He \rightarrow Ae$	pr2 ui
4	Ge	pr3 2 mp
5	Ae	pr3 3 mp
6	$Ae \wedge Ge$	4 5 adj
7	$\exists x(Ax \wedge Gx)$	6 eg dd

c. If some **H**yena is **G**rey, every hyena is grey
Every s**C**avenger is grey
J**e**nny is a hyena and a scavenger
K**a**thy is a hyena
∴ Kathy is grey

$\exists x(Hx \wedge Gx) \rightarrow \forall x(Hx \rightarrow Gx)$
$\forall x(Cx \rightarrow Gx)$
$He \wedge Ce$
Ha
∴ Ga

1	~~Show~~ Ga	
2	~~Show~~ $\exists x(Hx \wedge Gx)$	
3	He	pr3 s
4	Ce	pr3 s
5	$Ce \rightarrow Ge$	pr2 ui
6	Ge	4 5 mp
7	$He \wedge Ge$	3 6 adj
8	$\exists x(Hx \wedge Gx)$	7 eg dd
9	$\forall x(Hx \rightarrow Gx)$	2 pr1 mp
10	$Ha \rightarrow Ga$	9 ui
11	Ga	10 pr4 mp dd

2. The error is at line 3. It is not permissible to use EI to get an instance of pr2 in the variable z because z occurs already on line 2; this would violate the restriction on IE.

3. No derivations are given for named theorems.

SECTION 8

1. Symbolize these arguments and provide derivations to validate them. Give an explicit scheme of abbreviation for each.

a. If history is right (**P**), then if anyone was str**O**ng, **h**ercules was strong.
Only those who work out (**M**) are strong, and only those with self-**D**iscipline work out.
∴ If Hercules does not have self-discipline, then either history is not right or nobody is strong.

$P \rightarrow (\exists xOx \rightarrow Oh)$
$\forall x(Ox \rightarrow Mx) \wedge \forall x(Mx \rightarrow Dx)$
∴ $\sim Dh \rightarrow (\sim P \vee \sim\exists xOx)$

1	~~Show~~ ~Dh → (~P ∨ ~∃xOx)	
2	~Dh	ass cd
3	Mh → Dh	pr2 s ui
4	~Mh	2 3 mt
5	Oh → Mh	pr2 s ui
6	~Oh	4 5 mt
7	~~Show~~ ~P ∨ ~∃xOx	
8	~~Show~~ ~~P → ~∃xOx	
9	~~P	ass cd
10	P	9 dn
11	∃xOx → Oh	p1 10 mp
12	~∃xOx	6 11 mt cd
13	~P ∨ ~∃xOx	8 cdj dd
14		7 cd

b. If some **G**iraffes are not **H**appy, then all giraffes are **M**orose.
Some giraffes p**O**nder the mysteries of life.
∴ If some giraffes are not morose, then some who ponder the mysteries of life are happy.

∃x(Gx ∧ ~Hx) → ∀x(Gx → Mx)
∃x(Gx ∧ Ox)
∴ ∃x(Gx ∧ ~Mx) → ∃x(Ox ∧ Hx)

1	~~Show~~ ∃x(Gx ∧ ~Mx) → ∃x(Ox ∧ Hx)	
2	∃x(Gx ∧ ~Mx)	ass cd
3	Gi ∧ ~Mi	2 ei
4	~~Show~~ ~∀x(Gx → Mx)	
5	∀x(Gx → Mx)	ass id
6	Gi → Mi	5 ui
7	Mi	3 s 6 mp
8	~Mi	3 s 7 id
9	~∃x(Gx ∧ ~Hx)	4 pr1 mt
10	∀x~(Gx ∧ ~Hx)	9 qn
11	Gj ∧ Oj	pr2 ei
12	~(Gj ∧ ~Hj)	10 ui
13	~Gj ∨ ~~Hj	12 dm
14	Hj	11s dn 13 mtp dn
15	Oj ∧ Hj	11s 14 adj
16	∃x(Ox ∧ Hx)	15 eg cd

c. There is not a single **C**ritic who either **L**ikes art or can p**A**int.
Some level-**H**eaded pe**O**ple are critics.
Anyone who can't paint is un**E**ducated.
∴ Some level-headed people are uneducated.

∀x(Cx → ~(Lx ∨ Ax))
∃x((Hx ∧ Ox) ∧ Cx)
∀x(Ox → (~Ax → ~Ex))
∴ ∃x((Hx ∧ Ox) ∧ ~Ex)

1	~~Show~~ ∃x((Hx ∧ Ox) ∧ ~Ex)	
2	(Hi ∧ Oi) ∧ Ci	pr2 ei
3	Ci	2 s
4	Hi	2 s s
5	Oi	2 s s
6	Ci → ~(Li ∨ Ai)	pr1 ui
7	~(Li ∨ Ai)	3 6 mp
8	~Li ∧ ~Ai	7 dm
9	~Ai	8s
10	Oi → (~Ai → ~Ei)	pr3 ui
11	~Ai → ~Ei	5 10 mp
12	~Ei	9 11 mp
13	(Hi ∧ Oi) ∧ ~Ei	2 s 12 adj
14	∃x((Hx ∧ Ox) ∧ ~Ex)	13 eg dd

d. No **A**stronaut is a good **D**ancer.
Every s**I**nger is warm-**B**looded.
If something is warm-blooded and is not a good dancer, then nothing that is either a singer or an astronaut is **E**xultant.
∴ If some astronaut is a singer, then no singer is exultant.

∀x(Ax → ~Dx)
∀x(Ix → Bx)
∃x(Bx ∧ ~Dx) → ∀x((Ix ∨ Ax) → ~Ex)
∴ ∃x(Ax ∧ Ix) → ∀x(Ix → ~Ex)

1	~~Show~~ ∃x(Ax ∧ Ix) → ∀x(Ix → ~Ex)	
2	∃x(Ax ∧ Ix)	ass cd
3	~~Show~~ ∀x(Ix → ~Ex)	
4	~~Show~~ Ix → ~Ex	
5	Ix	ass cd
6	Ai ∧ Ii	2 ei
7	Ai → ~Di	pr1 ui
8	Ii → Bi	pr2 ui
9	~Di	6 s 7 mp
10	Bi ∧ ~Di	6s 8 mp 9 adj
11	∃x(Bx ∧ ~Dx)	10 eg
12	∀x((Ix ∨ Ax) → ~Ex)	11 pr3 mp
13	(Ix ∨ Ax) → ~Ex	12 ui
14	Ix ∨ Ax	5 add
15	~Ex	13 14 mp cd
16		4 ud
17		3 cd

e. All stu**D**ents who have a sense of **H**umor or are **B**rilliant seek **F**ame.
Anyone who seeks fame and is brilliant is **I**nsecure.
Whoever is a **M**athematician is brilliant.
∴ Every student who is a mathematician is insecure.

$\forall x((Dx \wedge (Hx \vee Bx)) \rightarrow Fx)$
$\forall x(Fx \wedge Bx \rightarrow Ix)$
$\forall x(Mx \rightarrow Bx)$
∴ $\forall x((Dx \wedge Mx) \rightarrow Ix)$

1	~~Show~~ $\forall x((Dx \wedge Mx) \rightarrow Ix)$	
2	~~Show~~ $(Dx \wedge Mx) \rightarrow Ix$	
3	$Dx \wedge Mx$	ass cd
4	Dx	3 s
5	Mx	3 s
6	$Mx \rightarrow Bx$	pr3 ui
7	Bx	5 6 mp
8	$Hx \vee Bx$	7 add
9	$Dx \wedge (Hx \vee Bx)$	4 8 adj
10	$(Dx \wedge (Hx \vee Bx)) \rightarrow Fx$	pr1 ui
11	Fx	9 10 mp
12	$Fx \wedge Bx$	7 11 adj
13	$Fx \wedge Bx \rightarrow Ix$	pr2 ui
14	Ix	12 13 mp cd
15		2 ud

f. There is a **M**onkey that is **H**appy if and only if some **G**iraffe is happy.
There is a monkey that is happy if and only if some giraffe is not happy.
All monkeys are happy.
∴ It is not the case that either every giraffe is happy or none are.

$\exists x(Mx \wedge (Hx \leftrightarrow \exists x(Gx \wedge Hx))$
$\exists x(Mx \wedge (Hx \leftrightarrow \exists x(Gx \wedge \sim Hx))$
$\forall x(Mx \rightarrow Hx)$
∴ $\sim(\forall x(Gx \rightarrow Hx) \vee \forall x(Gx \rightarrow \sim Hx))$

1	~~Show~~ $\sim(\forall x(Gx \rightarrow Hx) \vee \forall x(Gx \rightarrow \sim Hx))$	
2	$Mi \wedge (Hi \leftrightarrow \exists x(Gx \wedge Hx)$	pr1 ei
3	$Mj \wedge (Hj \leftrightarrow \exists x(Gx \wedge \sim Hx)$	pr2 ei
4	$Mi \rightarrow Hi$	pr3 ui
5	$Mj \rightarrow Hj$	pr3 ui
6	Hi	2 s 4 mp
7	Hj	3 s 5 mp
8	$\exists x(Gx \wedge Hx)$	2 s 6 adj eg
9	$\exists x(Gx \wedge \sim Hx)$	3 s 7 adj eg
10	$Gk \wedge Hk$	8 ei
11	$\sim\sim Hk$	10 s dn
12	$Gx \wedge \sim\sim Hk$	10 s 11 adj
13	$\sim(Gk \rightarrow \sim Hk)$	12 nc
14	$Gm \wedge \sim Hm$	9 ei
15	$\sim(Gm \rightarrow Hm)$	14 nc
16	$\exists x\sim(Gx \rightarrow \sim Hx)$	13 eg
17	$\sim\forall x(Gx \rightarrow \sim Hx)$	16 qn
18	$\exists x\sim(Gx \rightarrow Hx)$	15 eg
19	$\sim\forall x(Gx \rightarrow Hx)$	18 qn
20	$\sim\forall x(Gx \rightarrow Hx) \wedge \sim\forall x(Gx \rightarrow \sim Hx))$	17 19 adj
21	$\sim(\forall x(Gx \rightarrow Hx) \vee \forall x(Gx \rightarrow \sim Hx))$	20 dm dd

g. For every **A**stronaut that writes p**O**etry, there is one that doesn't.
For every astronaut that doesn't write poetry, there is one that does.
∴ If there are any astronauts, some write poetry and some don't.

∀x((Ax ∧ Ox) → ∃x(Ax ∧ ~Ox))
∀x((Ax ∧ ~Ox) → ∃x(Ax ∧ Ox))
∴ ∃xAx → ∃x(Ax ∧ Ox) ∧ ∃x(Ax ∧ ~Ox)

1	~~Show~~ ∃xAx → ∃x(Ax ∧ Ox) ∧ ∃x(Ax ∧ ~Ox)	
2	∃xAx	ass cd
3	Ai	2 ei
4	Oi ∨ ~Oi	T59
5	~~Show~~ Oi → ∃x(Ax ∧ Ox) ∧ ∃x(Ax ∧ ~Ox)	
6	Oi	ass cd
7	Ai ∧ Oi	3 6 adj
8	∃x(Ax ∧ Ox)	7 eg
9	Ai ∧ Oi → ∃x(Ax ∧ ~Ox)	Pr1 ui
10	∃x(Ax ∧ ~Ox)	7 9 mp
11	(∃x(Ax ∧ Ox) ∧ ∃x(Ax ∧ ~Ox))	8 10 adj cd
12	~~Show~~ ~Oi → ∃x(Ax ∧ Ox) ∧ ∃x(Ax ∧ ~Ox)	
13	~Oi	ass cd
14	Ai ∧ ~Oi	13 3 adj
15	∃x(Ax ∧ ~Ox)	14 eg
16	Ai ∧ ~Oi → ∃x(Ax ∧ Ox)	Pr2 ui
17	∃x(Ax ∧ Ox)	14 16 mp
18	(∃x(Ax ∧ Ox) ∧ ∃x(Ax ∧ ~Ox))	15 17 adj cd
19	∃x(Ax ∧ Ox) ∧ ∃x(Ax ∧ ~Ox)	4 5 12 sc
20		19 cd

<Could also skip line 4 and use sc appealing only to lines 5 and 12.>

SECTION 9

1. a. ~∃x(Ax ∨ Bx)
∀x∀y(Gx ∧ Hy → By)
∃xGx
∴ ∀x~Hx

1	~~Show~~ ∀x~Hx	
2	~∀x~Hx	ass id
3	∃xHx	2 qn
4	Hi	3 ei
5	Gj	pr3 ei
6	Gj ∧ Hi	4 5 adj
7	Gj ∧ Hi → Bi	pr2 ui ui
8	Bi	6 7 mp
9	∀x~(Ax ∨ Bx)	pr1 qn
10	~(Ai ∨ Bi)	9 ui
11	~Ai ∧ ~Bi	10 dm
12	~Bi	11 s 8 id

b. ∃x(Hx ∧ ~∃y(Gy ∧ Hx))
∴ ∀y~Gy

1	~~Show~~ ∀y~Gy	
2	~∀y~Gy	ass id
3	∃yGy	2 qn
4	Hi ∧ ~∃y(Gy ∧ Hi)	pr1 ei
5	Hi	4 s
6	~∃y(Gy ∧ Hi)	4 s
7	Gj	3 ei
8	Gj ∧ Hi	5 7 adj
9	∃y(Gy ∧ Hi)	8 eg 6 id

c. ∀x(Ax → ∀y(Bx ↔ By))
∃zBz
∴ ∀y(Ay → By)

1	~~Show~~ ∀y(Ay → By)	
2	~~Show~~ ∀i(Ai → Bi)	
3	~~Show~~ Ai → Bi	
4	Ai	ass cd
5	Ai → ∀y(Bi ↔ By)	pr1 ui
6	∀y(Bi ↔ By)	4 5 mp
7	Bj	pr2 ei
8	Bi ↔ Bj	6 ui
9	Bi	8 bc 7 mp cd
10		3 ud
11	∀y(Ay → By)	2 av dd

d. ~∀x(Dx ∨ Ex)
∃x(Fx ↔ ~Ex) → ∀zDz
∴ ∃x~Fx

1	~~Show~~ ∃x~Fx	
2	~∃x~Fx	ass id
3	∀xFx	2 qn
4	∃x~(Dx ∨ Ex)	pr1 qn
5	~(Di ∨ Ei)	4 ei
6	~Di ∧ ~Ei	5 dm
7	~Di	6 s
8	~Ei	6 s
9	~~Show~~ Fi → ~Ei	
10	~Ei	8 r cd
11	~~Show~~ ~Ei → Fi	
12	Fi	3 ui cd
13	Fi ↔ ~Ei	9 11 cb
14	∃x(Fx ↔ ~Ex)	13 eg
15	∀zDz	14 pr2 mp
16	Di	15 ui 7 id

e. $Jc \wedge {\sim}Jd$
$\forall xKx \vee \forall x{\sim}Kx$
$\exists x(Jx \wedge Kx) \rightarrow \forall x(Kx \rightarrow Jx)$
$\therefore\ {\sim}Kc$

1	~~Show~~ $\sim Kc$	
2	Kc	ass id
3	~~Show~~ $\sim\forall x{\sim}Kx$	
4	$\forall x{\sim}Kx$	ass id
5	$\sim Kc$	4 ui
6	Kc	2 r 5 id
7	$\forall xKx$	3 pr2 mtp
8	$Jc \wedge Kc$	pr1 s 2 adj
9	$\exists x(Jx \wedge Kx)$	8 eg
10	$\forall x(Kx \rightarrow Jx)$	9 pr3 mp
11	$Kd \rightarrow Jd$	10 ui
12	Kd	7 ui
13	Jd	11 12 mp
14	$\sim Jd$	pr1 s 13 id

SECTION 10

1. a. $\forall x(Ax \rightarrow \exists y(By \wedge {\sim}Ay))$
$\sim\forall xBx$
$\sim\exists x(Bx \wedge Cx)$
$\therefore\ \exists x(Ax \wedge Cx)$

Universe: {1, 2, 3}
A: {1}
B: {2}
C: {3}

b. $\exists x(Dx \wedge Ex \wedge {\sim}Fx)$
$\exists x({\sim}Dx \wedge {\sim}Ex)$
$\forall x(Ex \rightarrow Dx \vee Fx)$
$\therefore\ \forall x(Dx \wedge Ex \rightarrow {\sim}Fx)$

Universe: {1, 2, 3}
D: {1, 2}
E: {1, 2}
F: {1}

c. $\exists x(Fx \wedge Gx)$
$\exists x(Fx \wedge {\sim}Gx)$
$\exists x({\sim}Fx \wedge Gx)$
$\therefore\ \forall x({\sim}Fx \rightarrow Gx)$ <requires more than three things in the universe>

Universe: {1, 2, 3, 4}
F: {2, 3}
G: {1, 2}

d. $\forall x\exists y(Fx \leftrightarrow (Gy \vee Fx))$
$\therefore\ {\sim}\exists xFx \rightarrow {\sim}\exists xGx$

Universe: {1, 2}
F: { }
G: {1}

e. $Ha \land \sim Hb$
$\forall x(Kx \to Hx \land Jx)$
$\exists x(Jx \land \sim Kx)$
$\therefore\ \exists x(Hx \land \sim Jx)$

Universe: {1, 2}
H: {1}
J: {1,2}
K: { }
a --- 1
b --- 2

SECTION 11

1. For each of the following arguments use the method of expansions to determine whether the following is a counterexample for it or not.

Universe: ❶ ❷ ❸

F: {❶}
G: {❶, ❸}
H: {❸}
a: ❸
b: ❶

a. $\forall x(Hx \to \exists y(Fy \land \sim Hy))$
$\sim\forall xFx$
$\sim\exists x(Fx \land Gx)$
$\therefore\ \exists x(Hx \land Gx)$

The conclusion expands to:

$(H1 \land G1) \lor (H2 \land G2) \lor (\mathbf{H3} \land \mathbf{G3})$

which is true because H3 and G3 are true. Since we have a true conclusion, we don't have a counterexample.

b. $\exists x(Gx \land Hx \land \sim Fx)$
$\exists x(\sim Gx \land \sim Hx)$
$\forall x(Hx \to Gx \lor Fx)$
$\therefore\ \forall x(Gx \land Hx \to \sim Fx)$

The conclusion expands to:

$(G1 \land \mathbf{H1} \to \sim F1) \land (\mathbf{G2} \land \mathbf{H2} \to \sim F2) \land (G3 \land H3 \to \mathbf{\sim F3})$

which is true because the first conjunct has a false antecedent, the second conjunct has a false antecedent, and the third conjunct has a true consequent. Since we have a true conclusion, we don't have a counterexample.

c. $\exists x(Fx \land Gx)$
$\exists x(Fx \land \sim Gx)$
$\exists x(\sim Fx \land Gx)$
$\therefore\ \forall x(\sim Fx \to Gx)$

The second premise expands to:

$(F1 \land \sim G1) \lor (F2 \land \sim G2) \lor (F3 \land \sim G3)$

which is false because each disjunct is false. Since we have a false premise we don't have a counterexample.

d. $\forall x \exists y(Fx \leftrightarrow (Gy \vee Fx))$
$\therefore \sim\exists xFx \rightarrow \sim\exists xGx$

The conclusion expands to:

$$\sim(\mathbf{F1} \vee F2 \vee F3) \rightarrow \sim(G1 \vee G2 \vee G3)$$

which is true because the antecedent is false because its leftmost disjunct is true. Since we have a true conclusion we don't have a counterexample.

e. $Ha \wedge \sim Hb$
$\forall x(Fx \rightarrow Hx \wedge Gx)$
$\exists x(Gx \wedge \sim Fx)$
$\therefore \exists x(Hx \wedge \sim Gx)$

The second premise expands to:

$$(F1 \rightarrow \mathbf{H1} \wedge G1) \wedge (F2 \rightarrow H2 \wedge G2) \wedge (F3 \rightarrow H3 \wedge G3)$$

which is false because the first conjunct has a true antecedent and a false consequent. Since we have a false premise, we don't have a counterexample.