

1. Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of three successive vehicles.
  - a. List all outcomes in the event  $A$  that all three vehicles go in the same direction.
  - b. List all outcomes in the event  $B$  that all three vehicles take different directions.
  - c. List all outcomes in the event  $C$  that exactly two of the three vehicles turn right.
  - d. List all outcomes in the event  $D$  that exactly two vehicles go in the same direction.
  - e. List outcomes in  $D'$ ,  $C \cup D$ , and  $C \cap D$ .

**ANSWER:**

- a. Event  $A = \{ RRR, LLL, SSS \}$
- b. Event  $B = \{ RLS, RSL, LRS, LSR, SRL, SLR \}$
- c. Event  $C = \{ RRL, RRS, RLR, RSE, LRR, SRR \}$
- d. Event  $D = \{ RRL, RRS, RLR, RSE, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS \}$
- e. Event  $D'$  contains outcomes where all cars go the same direction, or they all go different directions:  
 $D' = \{ RRR, LLL, SSS, RLS, RSL, LRS, LSR, SRL, SLR \}$   
 Because Event  $D$  totally encloses Event  $C$ , the compound event  $C \cup D = D$ . Therefore,  $C \cup D = \{ RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS \}$   
 Using similar reasoning, we see that the compound event  $C \cap D = C$ . Therefore,  $C \cap D = \{ RRL, RRS, RLR, RSR, LRR, SRR \}$

2. A college library has five copies of a certain text on reserve. Two copies (1 and 2) are first printings, and the other three (3, 4, and 5) are second printings. A student examines these books in random order, stopping only when a second printing has been selected. One possible outcome is 4, and another is 125.
  - a. List the  $S$ .
  - b. Let  $A$  denote the event that exactly one book must be examined. What outcomes are in  $A$ ?
  - c. Let  $B$  be the event that book 4 is the one selected. What outcomes are in  $B$ ?
  - d. Let  $C$  be the event that book 2 is not examined. What outcomes are in  $C$ ?

**ANSWER:**

- a.

Outcome Number	Outcome
1	123
2	124
3	125
4	213
5	214
6	215
7	13
8	14
9	15
10	23
11	24
12	25
13	3
14	4
15	5

- b. Outcome numbers 13, 14, 15, so  $A = \{3, 4, 5\}$
- c. Outcome numbers 2, 5, 8, 11, 14 so  $B = \{124, 214, 14, 24, 4\}$

- d. Outcome numbers 7, 8, 9, 13, 14, 15 so  $C = \{13, 14, 15, 3, 4, 5\}$
3. A certain sports car comes equipped with either an automatic or a manual transmission, and the car is available in one of four colors. Relevant probabilities for various combinations of transmission type and color are given in the accompanying table.

Transmission Type	Color			
	White	Blue	Black	Red
A	.13	.10	.11	.11
M	.15	.07	.15	.18

Let  $A =$  (automatic transmission),  $B =$  {black}, and  $C =$  {white}.

- Calculate  $P(A)$ ,  $P(B)$ , and  $P(A \cap B)$ .
- Calculate both  $P(A|B)$  and  $P(B|A)$ , and explain in context what each of these probabilities represents.
- Calculate and interpret  $P(A|C)$  and  $P(A|C')$ .

**ANSWER:**

- $$P(A) = .13 + .10 + .11 + .11 = .45,$$

$$P(B) = .11 + .15 = .26$$

$$P(A \cap B) = .11$$

- $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.11}{.26} = .4231$$

Knowing that the car is black, the probability that it has an automatic transmission is .4231.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.11}{.45} = .2444$$

Knowing that the car has an automatic transmission, the probability that it is black is .2444.

- $$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{.13}{.28} = .4643$$

The probability that the car has automatic transmission, knowing that the car is white is .4643.

$$P(A|C') = \frac{P(A \cap C')}{P(C')} = \frac{.32}{.72} = .4444$$

Knowing that the car is not white, the probability that it has an automatic transmission is .4444.

4. Consider the following information: where  $A =$  {Visa Card},  $B =$  {MasterCard},  $P(A) = .5$ ,  $P(B) = .4$ , and  $P(A \cap B) = .25$ . Calculate each of the following probabilities.
- $P(B|A)$
  - $P(B'|A)$
  - $P(A|B)$
  - $P(A'|B)$
  - Given that an individual is selected at random and that he or she has at least one card, what is the probability that he or she has a Visa card?

**ANSWER:**

- $$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.25}{.50} = .50$$

$$\begin{aligned} \text{b. } P(B'|A) &= \frac{P(A \cap B')}{P(A)} = \frac{.25}{.50} = .50 \\ \text{c. } P(A|B) &= \frac{P(A \cap B)}{P(B)} = \frac{.25}{.40} = .6125 \\ \text{d. } P(A'|B) &= \frac{P(A' \cap B)}{P(B)} = \frac{.15}{.40} = .3875 \\ \text{e. } P(A|A \cup B) &= \frac{P[A \cap (A \cup B)]}{P(A \cup B)} = \frac{.50}{.65} = .7692 \end{aligned}$$

5. A certain shop repairs both audio and video components. Let A denote the event that the next component brought in for repair is an audio component, and let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose that  $P(A) = .625$  and  $P(B) = .05$ . What is  $P(B/A)$ ?

**ANSWER:**

$$\text{Since B is contained in A, } A \cap B = B, \text{ then } P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(B)}{P(A)} = \frac{.05}{.625} = .08$$

6. At a certain gas station, 40% of the customers use regular unleaded gas ( $A_1$ ), 35% use extra unleaded gas ( $A_2$ ), and 25% use premium unleaded gas ( $A_3$ ). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using extra gas, 60% fill their tanks, whereas of those using premium, 50% fill their tanks.
- What is the probability that the next customer will request extra unleaded gas and fill the tank?
  - What is the probability that the next customer fills the tank?
  - If the next customer fills the tank, what is the probability that regular gas is requested? Extra gas? Premium gas?

**ANSWER:**

$$P A_1 = .40, P A_2 = .35, P A_3 = .25$$

$$P B|A_1 = .30$$

$$P B|A_2 = .60$$

$$P B|A_3 = .50$$

Therefore,

$$P A_1 \cap B = P A_1 \cdot P B|A_1 = .40 \cdot .30 = .12$$

$$P A_2 \cap B = P A_2 \cdot P B|A_2 = .35 \cdot .60 = .21$$

$$P A_3 \cap B = P A_3 \cdot P B|A_3 = .25 \cdot .50 = .125$$

- $P(A_2 \cap B) = .21$
- $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .12 + .21 + .125 = .455$
- $P(P(A_1|B)) = \frac{P(A_1 \cap B)}{P(B)} = \frac{.12}{.455} = .264$   
 $P(A_2|B) = \frac{P(A_2 \cap B)}{P(B)} = \frac{.21}{.455} = .462$   
 $P(A_3|B) = \frac{P(A_3 \cap B)}{P(B)} = \frac{.125}{.455} = .275$

7. Two pumps connected in parallel fail independently of one another on any given day. The probability that only the older pump will fail is .15, and the probability that only the newer pump will fail is .05. What is the probability that the pumping system will fail on any given day (which happens if both pumps fail)?

**ANSWER:**

Let  $A_1$  = older pump fails,  $A_2$  = newer pump fails, and  $x = P(A_1 \cap A_2)$ . The  $P(A_1) = .15 + x$ ,  $P(A_2) = .05 + x$ , and  $x = P(A_1 \cap A_2) = P(A_1) \cdot P(A_2) = (.15 + x)(.05 + x)$ . The resulting quadratic equation,  $x^2 - .80x + .0075 = 0$ , has roots  $x = .0095$  and  $x = .7905$ . Hopefully the smaller root is the actual probability of system failure.

8. An insurance company offers its policyholders a number of different payment options. For a randomly selected policyholder, let  $X$  = the number of months between successive payments. The cdf of  $X$  is as follows:

$$F(x) = \begin{cases} 0 & x < 1 \\ .30 & 1 \leq x < 3 \\ .40 & 3 \leq x < 4 \\ .45 & 4 \leq x < 6 \\ .60 & 6 \leq x < 12 \\ 1 & 12 \leq x \end{cases}$$

- What is the pmf of  $X$ ?
- Using just the cdf, compute  $P(3 \leq X \leq 6)$  and  $P(X \geq 4)$ .
- Using just the pmf, compute  $P(X > 6)$ .

**ANSWER:**

- Possible  $X$  values are those values at which  $F(x)$  jumps, and the probability of any particular value is the size of the jump at that value. Thus we have:

$x$	1	3	4	6	12
$P(x)$	.30	.10	.05	.15	.40

- $P(3 \leq X \leq 6) = F(6) - F(3-) = .60 - .30 = .30$ ,  
 $P(X \geq 4) = 1 - P(X < 4) = 1 - F(4-) = 1 - .40 = .60$
- $P(X > 6) = P(X = 12) = .40$

9. The pmf for  $X$  = the number of major defects on a randomly selected gas stove of a certain type is

$x$	0	1	2	3	4
$P(x)$	.10	.15	.45	.25	.05

Compute the following:

- $E(X)$
- $V(X)$  directly from the definition
- The standard deviation of  $X$
- $V(X)$  using the shortcut formula

**ANSWER:**

- $E(X) = \sum_{x=0}^4 x \cdot p(x) = (0)(.10) + (1)(.15) + (2)(.45) + (3)(.25) + (4)(.05) = 2.0$

- b.  $V(X) = \sum_{x=0}^4 (x-2.0)^2 \cdot p(x) = (0-2.0)^2(.10) + \dots + (4-2.0)^2(.05) = .40 + .15 + 0.0 + .25 + .20 = 1.0$
- c.  $\sigma_x = \sqrt{1.0} = 1.0$
- d.  $V(X) = [\sum_{x=0}^4 x^2 \cdot p(x)] - (2.0)^2 = 5.0 - 4.0 = 1.0$

10. An appliance dealer sells three different models of upright freezers having 13.5, 15.9, and 19.1 cubic feet of storage space, respectively. Let  $X$  = the amount of storage space purchased by the next customer to buy a freezer. Suppose that  $X$  has pmf

$x$	13.5	15.9	19.1
$P(x)$	.2	.4	.4

- a. Compute  $E(X)$ ,  $E(x^2)$ , and  $V(X)$ .
- b. If the price of a freezer having capacity  $X$  cubic feet is  $25X - 8.5$ , what is the expected price paid by the next customer to buy a freezer?
- c. What is the variance of the price  $25X - 8.5$  paid by the next customer?
- d. Suppose that although the rated capacity of a freezer is  $X$ , the actual capacity is  $h(X) = X - .01X^2$ . What is the expected actual capacity of the freezer purchased by the next customer?

**ANSWER:**

- a.  $E(X) = (13.5)(.2) + (15.9)(.4) + (19.1)(.4) = 16.70$ ,  
 $E(X^2) = (13.5)^2(.2) + (15.9)^2(.4) + (19.1)^2(.4) = 283.498$ ,  
 $V(X) = 283.498 - (16.70)^2 = 4.608$
- b.  $E(25X - 8.5) = 25E(X) - 8.5 = (25)(16.70) - 8.5 = 409$
- c.  $V(25X - 8.5) = V(25X) = (25)^2V(X) = (625)(4.608) = 2880$
- d.  $E[h(X)] = E[X - .01X^2] = E(X) - .01E(X^2) = 16.7 - 2.835 = 13.865$

11. Suppose that only 25% of all drivers come to a complete stop at an intersection having flashing red lights in all directions when no other cars are visible. What is the probability that, of 20 randomly chosen drivers coming to an intersection under these conditions,
- a. At most 6 will come to a complete stop?
- b. Exactly 6 will come to a complete stop?
- c. At least 6 will come to a complete stop?
- d. How many of the next 20 drivers do you expect to come to a complete stop?

**ANSWER:**

Let  $S$  = comes to a complete stop, so  $p = .2$ ,  $n = 20$

- a.  $P(X \leq 6) = B(6; 20, .25) = .786$
- b.  $P(X = 6) = b(6; 20, .25) = B(6; 20, .25) - B(5; 20, .25) = .786 - .617 = .169$
- c.  $P(X \geq 6) = 1 - P(X \leq 5) = 1 - B(5; 20, .25) = 1 - .617 = .383$
- d.  $E(X) = (20)(.25) = 5$ . We expect 5 of the next 20 to stop.
12. A geologist has collected 10 specimens of basaltic rock and 10 specimens of granite. The geologist instructs a laboratory assistant to randomly select 15 of the specimens for analysis.
- a. What is the pmf of the number of granite specimens selected for analysis?
- b. What is the probability that all specimens of one of the two types of rock are selected for analysis?
- c. What is the probability that the number of granite specimens selected for analysis is within 1 standard deviation of its mean value?

**ANSWER:**

- a. Possible values of  $X$  are 5,6,7,8,9,10. (In order to have less than 5 of the granite, there would have to be more than 10 of the basaltic).

$$P(X = 5) = h(5; 15, 10, 20) = \frac{\binom{10}{5} \binom{10}{10}}{\binom{20}{15}} = .0163.$$

Following the same pattern for the other values, we arrive at the pmf, in table form below.

$x$	5	6	7	8	9	10
$P(x)$	.0163	.1354	.3483	.3483	.1354	.0163

- b.  $P(\text{all 10 of one kind or the other}) = P(X = 5) + P(X = 10) = .0163 + .0163 = .0326$   
 c.  $E(X) = n \cdot (M/N) = 15(10/20) = 7.5$ ;  
 $V(X) = (5/19)(7.5)[1 - (10/20)] = .9869$ ;  $\sigma_x = .9934$   
 $\mu \pm \sigma = 7.5 \pm .9934 = (6.5066, 8.4934)$ , so we want  
 $P(X = 7) + P(X = 8) = .3483 + .3483 = .6966$

13. Compute the following binomial probabilities directly from the formula for  $b(x; n, p)$ .

- a.  $b(3; 8, .7)$   
 b.  $b(5; 8, .7)$   
 c.  $P(3 \leq X \leq 5)$  when  $n = 8$  and  $p = .7$   
 d.  $P(X \geq 1)$  when  $n = 10$  and  $p = .1$

**ANSWER:**

- a.  $b(3; 8, .7) = \binom{8}{3} (.7)^3 (.3)^5 = (56)(.00083349) = .0467$   
 b.  $b(5; 8, .7) = \binom{8}{5} (.7)^5 (.3)^3 = (56)(.00453789) = .2541$   
 c.  $P(3 \leq X \leq 5) = b(3; 8, .6) + b(4; 8, .6) + b(5; 8, .6) = .0467 + .1361 + .2541 = .4369$   
 d.  $P(X \geq 1) = 1 - P(X = 0) = 1 - \binom{10}{0} (.1)^0 (.9)^{10} = 1 - (.9)^{10} = .6513$

14. Use the cumulative binomial probabilities table available in your text to obtain the following probabilities:

- a.  $B(4; 10, .4)$   
 b.  $b(4; 10, .4)$   
 c.  $b(6; 10, .6)$   
 d.  $P(2 \leq X \leq 4)$  when  $X \sim \text{Bin}(10, .4)$   
 e.  $P(2 \leq X)$  when  $X \sim \text{Bin}(10, .3)$   
 f.  $P(X \leq 1)$  when  $X \sim \text{Bin}(10, .7)$   
 g.  $P(2 < X < 6)$  when  $X \sim \text{Bin}(10, .3)$

**ANSWER:**

- a.  $B(4; 10, .4) = .633$   
 b.  $b(4; 10, .4) = B(4; 10, .4) - B(3; 10, .4) = .251$

- c.  $b(6;10,.6) = B(6;10,.6) - B(5;10,.6) = .251$
- d.  $P(2 \leq X \leq 4) = B(4;10,.4) - B(1;10,.4) = .587$
- e.  $P(2 < X) = 1 - P(X \leq 1) = 1 - B(1;10,.3) = .851$
- f.  $P(X \leq 1) = B(1;10,.7) = .0000$
- g.  $P(2 < X < 6) = P(3 \leq X \leq 5) = B(5;10,.3) - B(2;10,.3) = .570$