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James P. Feehan and Raymond G. Batina

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Labor and Capital Taxation with Public Inputs as Common Property

James P. Feehan

Memorial University of Newfoundland, Canada

Raymond G. Batina

Washington State University

The services of many public inputs (e.g., dams, irrigation systems, and highways) are provided to private firms on a free-access basis. If these services enter constant-returns-to-scale production functions then there are decreasing returns to scale in the private factors. Thus a change in the amount of a public input gives rise to positive rent or economic profit in the first instance. The authors extend the literature by recognizing that this rent cannot be an equilibrium phenomenon. Private agents will engage in rent-seeking that will ultimately lead to dissipation. This makes a public input equivalent to a common property resource, which, in the absence of the appropriate price or quantity rationing, gives rise to inefficiency. Using a model with capital and labor as private inputs, the authors show it is optimal to tax capital even though a labor tax is available and capital is internationally mobile. Production efficiency also holds since our policy supports the first-best equilibrium despite decreasing returns to scale in private inputs.

Keywords: *common property; rent-dissipation; taxation of capital; production efficiency*

1. Introduction

Public inputs such as basic scientific research and public infrastructure are generally recognized as enhancing an economy's production possibilities for private goods.¹ This is succinctly captured by a production function where the output of a private good is a function of private inputs and a public input. Two important cases of public inputs have been the focus of much recent research.² Some researchers have considered the case where there are constant returns to scale (CRS) in private inputs alone. A

proportionate increase in private inputs leads to a proportionate increase in the output of a private good in this case, without need for greater provision of any public inputs. Romer (1986), Barro (1990), and more recently, Haughwout (2002) and Feehan and Matsumoto (2000, 2002) provide examples. Following the terminology of Meade (1952), such public inputs are often described as "atmospheric" but have also been termed as *pure* or *factor-augmenting* in the more recent literature.

The second case, which is the subject of this article, arises when there is CRS in all inputs including the public input. An equiproportionate increase in private inputs requires a similar increase in the public input to increase output of the private good in the same proportion. Public inputs of this type are typically referred to as *unpaid factors*, and, unlike atmospheric public inputs, are subject to congestion in use; that is, there are decreasing returns (DRS) in private inputs.

Abstracting from public inputs, it is well known that DRS is problematic for production efficiency because DRS gives rise to economic profits that must be taxed away to allow production efficiency.³ Stiglitz and Dasgupta (1971) and Munk (1980) made some important early contributions on DRS, profits, and their impact on the optimal tax design and production efficiency. A more recent strand of that literature has considered DRS when capital is the sole input and it is mobile across jurisdictions. Huizinga and Nielson (1997) study the optimal taxation of capital when there are profits and foreign ownership of domestic firms in a model of a small, open economy. Source-based capital taxes are suboptimal when profits are fully taxed. However, source-based taxes are appropriate when profits are not fully taxed and are increasing in foreign ownership. Keen and Kotsogiannis (2002, 2004) study horizontal and vertical tax competition, and derive conditions under which one dominates the other when profits are taxed and when they are not. They also show that, regardless of which one dominates, adding more locations, thus increasing competition, actually exacerbates both problems.

Turning to the literature in which there is a public input and DRS in private inputs, an important early contribution was made by Pestieau (1976). Assuming a fixed number of firms, he allowed profits to accrue to

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the firm-owners in equilibrium, and investigated the design of commodity taxes and the rule for provision of the public input when profits could not be completely taxed away. Keen and Marchand (1997) study the optimal composition of government spending in a model of tax competition in which there is also a public input and DRS in private inputs. They assume that positive economic profits are realized in equilibrium but that there is a limitation on use of profit taxes. They find that tax competition leads to a bias in spending toward productive public inputs and away from public goods that confer consumption benefits. More recently, Dahlby and Wilson (2003) study vertical fiscal externalities using a model in which there is a public input and DRS in labor, the sole private input. They consider the case where a local government and a federal government share a tax base that includes labor income and profit income.

In this article, we take a different tact from the DRS-public input literature cited above. We do not assume that there are a fixed number of firms and other restrictions that allow profits to be realized in equilibrium. Rather, following Boadway (1973) and Henderson (1974), we recognize that such profits will be sought after by economic agents. If a public input of the unpaid-factor variety is made freely available, it will initially generate economic profit but such profit cannot persist. Firms, both existing and new entrants attracted by profits, will hire more of the private inputs or factors to capture those profits. This will continue until any potential economic profit is exhausted. The result, as predicted by Henderson (1974), is an inefficient use of private inputs, just as in the case of a common property resource.⁴ It also means that a profits tax, by itself, is ineffective since it raises no revenue.

We also study the optimal factor taxes needed to finance spending on the public input. We show that it is optimal to tax both capital and labor, even when capital is perfectly mobile internationally. The taxes raise revenue to finance the public input, and they also serve a Pigovian purpose in providing firms with an incentive to hire factors efficiently at the margin, thereby alleviating the “commons” problem. We also show that production efficiency prevails once the optimal taxes are imposed.

It is, of course, an empirical issue as to whether a particular public input is of one type or the other. Some public inputs are atmospheric, in the sense of Meade (1952). For instance, agricultural research and development (R&D) is likely to be a pure public input. Doubling private factors employed in the agricultural sector, with the same level of R&D, is likely to lead to a doubling of agricultural output. On the other hand, many forms of public services to industry (e.g., public infrastructure) are akin to

unpaid factors, especially when congestion occurs.⁵ Rationing these services on a per-firm basis or charging user fees for them is often difficult or not feasible, making the case studied in this article of some importance.

In the next section, we make the case that an unpaid factor type of public input is equivalent to a common property resource. In section 3 we explore the issue of optimal factor-tax financing in a standard two-factor model, taking explicit account of the common-property attribute of the public input. We show that both private inputs, labor and capital, must be taxed even though capital is perfectly mobile internationally. We also show that production efficiency will hold when the optimal taxes are imposed. Section 4 concludes the article.

2. A Public Input As Common Property

2.1. Rent Dissipation With One Private Factor

To set the stage for the discussion, it is convenient to briefly recall the commons problem. Consider the simple context in which there is just one final commodity, the quantity of which is denoted by Y . It is produced with one primary input, labor, and a common-property resource, the quantities of which are denoted by L and G , respectively. Assume that production function $F(\cdot)$ is linearly homogeneous in both inputs so $Y = F(L, G)$. By Euler's theorem it follows that $Y = F_L L + F_G G$, where subscripts denote partial derivatives so $F_L(F_G)$ is the marginal product of labor (the common-property resource). Both marginal products are positive, with $F_{LL} < 0$, $F_{GG} < 0$, and $F_{GL} > 0$. Firms, facing an output price P (normalized to unity) and a wage w for labor choose L , given an exogenous amount of G .

In this setting, efficiency requires that labor be hired up to where $F_L = w$. However, with free access to G , this condition cannot be realized. Recalling the Euler relation, at the efficient level of employment profit is $Y - wL = F_L L + F_G G - wL = F_G G > 0$. This profit will lead to more entry and increasing output. The outcome is that firms will hire up to the average product of labor (APL) equals the wage. Thus,

$$F_L + F_G G/L = w, \quad (1)$$

where $F_G G/L$ is the contribution of the resource to output expressed on a per-worker basis. Thus, an excess amount of labor is hired. This result occurs up to where profit is eliminated, or $Y - wL = F_L L + F_G G - wL = 0$, which results from the increased employment of labor and an

increased wage if the supply of labor is not perfectly elastic. This is the standard commons equilibrium.

It is also usual to assume that the APL is decreasing in L and increasing in G , so that $\partial(F_L + F_G G/L)/\partial L \equiv F_{LL} - F_G G/L^2 + F_{GL} G/L < 0$ and $\partial(F_L + F_G G/L)/\partial G \equiv F_{LG} + F_G/L + F_{GG} G/L > 0$.⁶ In light of the concavity properties of the production function, these widely accepted assumptions are not onerous.

2.2. Rent Dissipation With Two Private Factors

Next, consider how to extend the preceding results to a setting in which there are two primary factors of production. Output is now given by $Y = F(L, K, G)$, where K is private capital. Other than including that additional input, the production function is assumed to have the same characteristics as before: linear homogeneity in all inputs, positive decreasing marginal products, and so on.

There is an exogenous amount of G available on a free-access basis. Therefore, at a given wage w and rental r for capital, firms will hire the two primary factors in a cost-minimizing fashion. Since the production function is homogeneous, the capital-labor ratio will be determined solely by the wage-rental ratio.⁷ Consequently, the share of total income accruing to labor, $S = wL/(wL + rK)$, is determined and is solely a function of the capital-labor ratio. A similar result holds for capital's share, $1 - S$. Hence, the share of the contribution to output from the public input, $F_G G$, that goes to labor income must be S , with $(1 - S)$ going to capital income.

Thus, at the zero-profit equilibrium, the following conditions hold:

$$F_L + SF_G G/L = w, \quad (2)$$

and

$$F_K + (1 - S)F_G G/K = r. \quad (3)$$

Note that, if labor is the only primary factor, then $S = 1$ and equations (2) and (3) reduce to the standard one-factor result as given by (1) where labor would be hired up to where its average product equals the wage. Also recall from the one-factor case that the average product of labor decreases with L and increases with G . Letting the left-hand-sides of (2) and (3) be denoted by B and Z , respectively, the parallel assumptions for the two-factor case are $B_L < 0$, $B_G > 0$, $Z_K < 0$ and $Z_G > 0$. Moreover, we now have the cross-effects of B_K and Z_L , both of which are assumed to take positive values. (Expressions for these terms are derived in Appendix A for later discussion.)

To elucidate the two-factor equilibrium, it is useful to consider a specific case. Consider, a Cobb-Douglas production function of the form $Y = AK^\alpha L^\beta G^{(1-\alpha-\beta)}$. The efficient choices of capital and labor are found by equating each factor's price with the corresponding values of its marginal product. Those quantities are given by:

$$L^* = G[A(\alpha/r)^\alpha (\beta/w)^{(1-\alpha)}]^{1/(1-\alpha-\beta)}, \tag{4a}$$

and

$$K^* = G[A(\alpha/r)^{(1-\beta)} (\beta/w)^\beta]^{1/(1-\alpha-\beta)}, \tag{4b}$$

which yield a rent of $Y - wL - rK = (1 - \alpha - \beta)G[A(\alpha/r)^\alpha (\beta/w)^\beta]^{1/(1-\alpha-\beta)} > 0$. Consider instead the zero-profit outcome. Facing the same factor prices, firms would continue to choose the same capital-labor ratio as in the efficient case but would hire more capital and labor in an, ultimately fruitless, effort to capture profits. Those rent-dissipating employment levels are given by

$$L^{**} = [1/(\alpha + \beta)]^{1/(1-\alpha-\beta)} L^*, \tag{5a}$$

and

$$K^{**} = [1/(\alpha + \beta)]^{1/(1-\alpha-\beta)} K^*. \tag{5b}$$

Since $[1/(\alpha + \beta)]^{1/(1-\alpha-\beta)} > 1$, one can confirm that $K^{**} > K^*$ and $L^{**} > L^*$. Evaluating the marginal products, F_L, F_K , and F_G , at L^{**} and K^{**} , and noting that $S = \beta/(\alpha + \beta)$, one may also substitute into the left-hand sides of (2) and (3) to confirm their equality with the factor prices.

As demonstrated above, in the Cobb-Douglas case, S is a constant. This is not the case with other homogeneous production functions. For instance, consider a function characterized by constant elasticity of substitution, $Y = A[\alpha K^{-\eta} + \beta L^{-\eta} + (1 - \alpha - \beta)G^{-\eta}]^{(-1/\eta)}$, where η is the elasticity of substitution. From the first-order conditions for cost minimization, it follows that labor's share of total factor income is given by $S = \beta/[\alpha(k)^{-\eta} + \beta]$, where $k = K/L$ is the capital-labor ratio. Thus, in general, $S = S(k)$ for homogeneous production technologies.⁸ It then follows that $S_K = S_k/L, S_L = -kS_k/L$ and thus $S_L = -kS_K$.

Finally, reinterpret G as the quantity of a public input, available to private industry without charge or rationing. Firms will treat it just like any other common property resource. It is different from such a resource in that it is produced at a cost and its quantity is determined by public spending policy, but that difference is irrelevant to firms' behavior once it is made available.

2.3. Implications

The policy implications of free access to a public input are substantial and are analogous to those of a common property resource. The argument that profit-seeking firms will increase their use of private inputs to capture rents from a public input helps explain why there is increasing congestion on roads, highways, bridges, and airports, and why there are requests for more capacity to be added to these and other public facilities.

Starting in an equilibrium position, an increase in the public input will initially lead to economic profit when the public input is of the unpaid factor type. Firms react by increasing their demands for primary inputs, resulting in more employment of those factors and/or higher factor prices. This will continue up to where factor payments once again exhaust firms' revenue. Thus, building new infrastructure, for example, will not necessarily solve the problem of congestion, or the perceived inadequate level of infrastructure, in the long run. Even if it does "solve" the congestion problem, it is an inefficient solution since the level of public input would be excessive.

A first-best approach for achieving efficiency is to charge each firm for its utilization of the public input according to the value of its marginal product to the firm, as in Sandmo (1972), which is also the standard solution to the problem of the commons. No rent would be available and firms would revert to hiring the primary factors according to the usual conditions that equate the value of marginal product to the appropriate factor price. This option, however, may be difficult to implement in many cases since exclusion may be either impossible or prohibitively costly. Alternatively, to achieve a first-best outcome, the authorities could ration fixed amounts of the public input to a fixed number of firms, which is analogous to an individual quota system in a fishery. Firms would then realize profits in equilibrium. Such rents would also provide an attractive tax source for raising funds to finance the public input. Again, however, for many types of public inputs, rationing in this fashion is not practical.

According to the Coase Theorem, in the absence of any government rationing, firms themselves have an incentive to cooperate to realize and share the potential rent. Yet, despite that potential benefit, if there are a large number of firms in the industry or if there is free entry, then the chances of the firms cooperating would be severely limited.

Thus, the authorities not only have to provide the public input but also must deal with the consequent commons problem. Without some rationing mechanism, rent-seeking factor allocation occurs, and the economy fails

to achieve the first-best equilibrium even if a lump-sum tax or a profits tax is available. Other methods and revenue-raising instruments are needed to deal with the inefficiency arising from the rent dissipation problem. One such set of instruments is factor taxes. The design of such taxes is the subject matter of the remainder of this article.

3. Policy Design

3.1. The Model

To explore the implications for financing the public input when agents compete for profits, we study the following model. There are one private good, two private inputs, labor and capital, and one public input. The private inputs and the public input are used to produce the private good. As in Keen and Marchand (1997), we adopt the simplifying assumption that the marginal rate of transformation between the private good and the public input is a constant; one unit of the private good can be converted into $1/q$ units of the public input.⁹ The price of the private good is normalized to unity. The public input is produced efficiently so its price is given by q , its marginal cost.

The economy is small relative to the rest of the world, and capital is perfectly mobile while labor is not. Markets are perfectly competitive. Since the economy is small, the world interest rate is taken as exogenous. Many identical firms produce the goods, and consumers are identical as well, which allows us to consider the decisions of the representative agent.

The preferences of the representative consumer are given by a well-defined utility function of the form $U(C, E)$, where C is consumption of the private good and E is labor supply. Utility is increasing in consumption, decreasing in labor supply, quasi-concave in consumption, and quasi-convex in labor supply. The representative household consumes the private good subject to the following budget constraint:

$$C = (w - t)E + iK^+ \quad (6)$$

where w is the gross wage, t is a labor tax so $(w - t)$ is the net wage, K^+ is the quantity of domestically owned capital (assumed fixed), and i is the after-tax return on capital that can be realized on world markets, assumed to be exogenous. Let $r = i + T$ be the gross return on capital in this economy, where T is the tax on capital employed within the economy.

The household maximizes utility subject to its budget constraint. The associated demand for the private good and supply of labor are given by

$C = C(w - t, iK^+)$ and $E = E(w - t, iK^+)$, respectively. We assume substitution effects dominate so that labor supply is increasing in the net wage rate. It is straightforward to show that this implies that consumption is also increasing in the net wage. We assume leisure is a normal good; it follows from this that consumption is a normal good. Since the interest rate only causes an income effect for the consumer, it also follows that labor supply is decreasing in the net interest rate and consumption is increasing in the net interest rate. We substitute the consumption and labor supply functions into the utility function to obtain the indirect utility function, $V(w - t, iK^+)$, which has the usual derivative properties.

Next, turn to the hiring decisions of firms. From our earlier discussion, firms hire labor and capital according to (2) and (3). These conditions imply that the demands for labor and capital are functions of w, r , and G , written as $K(w, r, G)$ and $L(w, r, G)$, respectively. We derive the comparative statics of these private input demands in Appendix A. It is shown there that the factor demands for private inputs are negatively related to factor prices (i.e., $L_w < 0, L_r < 0, K_w < 0$, and $K_r < 0$), and the responses to the public input are positive (i.e., $L_G > 0$ and $K_G > 0$). Note that the signs of L_r and K_w are negative. Thus, the negative output effect of a cross-price change dominates the associated positive substitution effect.

We must also specify the government's behavior. The instruments available to finance the public input are the taxes on the employment of labor and capital, t and T , respectively. Note that a profits tax on firms, even if available, would not yield any revenue in equilibrium in our model because profits are dissipated by entry. The government chooses policy to maximize indirect utility subject to its budget constraint, which is given by

$$tE + TK = qG, \quad (7)$$

where K is the entire amount of capital located within the small economy, whether domestically owned or not. The supply of capital is perfectly elastic at the world net return.

Finally, to close the model, we include a labor market equilibrium condition. This is given by the equality of supply with demand:

$$E(w - t, iK^+) = L(w, i + T, G). \quad (8)$$

By totally differentiating this condition and using the comparative statics results of the factor demands, it is straightforward to show that the gross wage is increasing in the tax on labor, decreasing in the tax on capital, and increasing in the public input, but responds ambiguously to the net interest rate.¹⁰

3.2. Main Result

The government chooses its policy (t, T, G) to maximize the indirect utility function of the representative agent subject to its budget constraint and taking into account the response of the labor market equilibrium to the policy. Thus, the optimization problem may be expressed as the following Lagrangean:

$$H = V(w - t, iK^+) + a[tL(w, i + T, G) + TK(w, i + T, G) - qG] + b[L(w, i + T, G) - E(w - t, iK^+)] \quad (9)$$

where a is the lagrange multiplier on the government's budget constraint and b is the multiplier for the labor-market clearing condition.

Appendix B shows that the maximization of (9) over the four choice variables $(G, t, T, \text{ and } w)$ yields the following policy regime:¹¹

$$T = (1 - S)F_G G / K, \quad (10)$$

$$t = S F_G G / L, \quad (11)$$

$$F_G = q. \quad (12)$$

The intuition behind these results is straightforward. Essentially, when the government provides a public input that generates positive rents, private agents hire more private inputs to produce additional output to capture those extra profits. This creates the problem of the commons vis-à-vis factor markets and leads to inefficient (over) use of inputs. The optimal taxes on capital and labor in (10) and (11), respectively, are designed to eliminate this incentive. Once this incentive is removed, the public input should be provided according to the first-best rule; that is, it should be increased until the marginal benefit coincides with its marginal cost. Indeed, equation (12) indicates that the provision of the public input should be determined by the first-best rule, as in Kaizuka (1965), so that the equilibrium is characterized by production efficiency. This result is analogous to the finding by Feehan and Matsumoto (2002), who find that with an atmospheric public input, if factor taxes are set optimally, then this type of public input should also be provided according to the first-best rule. Despite the distortionary effect of the factor taxes, production efficiency remains desirable, a result consistent with Diamond and Mirrlees (1971a, 1971b).

The tax on capital is proportional to the public input's contribution to output per unit of capital. Thus, it is optimal to tax capital despite the international mobility of capital in our model. Otherwise, an increase in G

leads to an increased demand for capital causing an excessive inflow of foreign capital into the economy. In addition, the taxation of capital does not disrupt the production efficiency result either. Similarly, the tax rate on labor is proportional to the public input's contribution to output per unit of labor. A tax on labor should be employed even though labor supply is endogenous. Of course, the extent to which the labor tax and the capital tax are proportional to the public input's contribution to output must exactly match the extent to which the public input would have otherwise induced excessive employment of each factor.

These three key results, the labor tax, the capital tax, and the spending condition, indicate that the first-best optimum is achievable if factor taxes are designed appropriately. The factor taxes serve dual purposes. They raise revenue to finance spending on the public input, and they correct the distortion that otherwise occurs as a result of making the public services available to industry without charge. In short, adopting factor taxes in the correct proportions is a perfect substitute for charging directly for the use of the public input in our model, and consequently, this tax policy can support a first-best equilibrium.

4. Conclusion

Many forms of public inputs are of the unpaid-factor variety. We demonstrated that there is an equivalence between an unpaid factor type of public input and a common property resource. Analyses involving these types of public input need to address those common-property characteristics.

The key contribution of this article is to explicitly incorporate these common-property features in deriving the optimal design of factor taxes that can be employed to finance a public input.¹² The tax on each factor is proportional to the public input's contribution to total output per unit of the respective factor. Moreover, these taxes are of a Pigovian nature, and thus support a first-best efficiency outcome. Second-best considerations do not apply. Hence, even, or perhaps especially, if capital is completely mobile internationally, then a tax on capital is still called for because the provision of the public input would lead to an excessive inflow of capital from abroad, or too small an outflow.

Finally, in the absence of these factor taxes, or their equivalents, it would be difficult to assess whether congested public infrastructure, or demands by industry for more supportive government services, are signals of inadequate supply or are simply the result of excessive use due to free

access. Indeed, increasing congestion could very well be a signal that private agents are seeking extra profits due to improved infrastructure.

Appendix A

Start by taking the total differentials of (2) and (3) with respect to L, K, G, w , and r . The result may be expressed as

$$B_L dL + B_K dK = dw - B_G dG \tag{A1}$$

and

$$Z_L dL + Z_K dK = dr - Z_G dG, \tag{A2}$$

where, as in the text, B_K, B_G, Z_L , and Z_G are positive while B_L and Z_K are negative. The expressions for these terms are as follows:

$$B_L = [F_{LL} + SF_{GL}G/L + S_L F_G G/L - SF_G G/L^2], \tag{A3}$$

$$B_K = [F_{LK} + SF_{GK}G/L + S_K F_G G/L], \tag{A4}$$

$$B_G = [F_{GL} + SF_{GG}G/L + SF_G/L], \tag{A5}$$

$$Z_L = [F_{KL} + (1 - S)F_{GL}G/K - S_L F_G G/K], \tag{A6}$$

$$Z_K = [F_{KK} + (1 - S)F_{GK}G/K - S_K F_G G/K - (1 - S)F_G G/K^2], \tag{A7}$$

$$Z_G = [F_{GK} + (1 - S)F_{GG}G/K + (1 - S)F_G/K]. \tag{A8}$$

Next, from the linear homogeneity of the production function, it follows that

$$F_{LL}L + F_{KL}K + F_{GL}G = 0, \tag{A9}$$

$$F_{LK}L + F_{KK}K + F_{GK}G = 0, \tag{A10}$$

$$F_{LG}L + F_{KG}K + F_{GG}G = 0. \tag{A11}$$

Recall that $S_K = -LS_L/K$. Substitute for S_K in (A4) and (A7). Use (A9) and (A10), respectively, to substitute for $F_{GL}G$ and $F_{GK}G$ in (A3), (A4), (A6), and (A7). Also, use (A11) to substitute for F_{GL} in (A5). Then substitution of those results into (A1) and (A2) yields the following:

$$[(-K/L)(Z_L + SF_G G/LK)]dL + [B_K]dK = dw - [B_G]dG \tag{A12}$$

$$[Z_L]dL + [-LB_K/K - (1 - S)F_G G/K^2]dK = dr - [-LB_G/K + F_G/K]dG \tag{A13}$$

where, following those substitutions,

$$B_K = (1 - S)F_{LK} - SF_{KK}K/L - S_L F_G G/K$$

$$Z_L = -(1 - S)F_{LL}L/K + SF_{KL} + S_L F_G G/K$$

$$B_G = -[F_{GK}K + (1 - S)F_{GG}G - SF_G]/L$$

Next, the determinant of the matrix formed by the left-hand sides of (A12) and (A13) is

$$D = (F_G G / LK) [S B_K + (1 - S) Z_L + S(1 - S) F_G G / LK]. \quad (\text{A14})$$

Since B_K and Z_L are positive, $D > 0$.

Applying Cramer's rule to (A12) and (A13) then yields:

$$L_w = [(-L/K)B_K - (1 - S)F_G G / K^2] / D < 0,$$

$$L_r = -B_K / D < 0,$$

$$K_w = -Z_L / D < 0,$$

$$K_r = [(-K/L)Z_L - S F_G G / L^2] / D < 0,$$

$$L_G = (F_G / K) [B_K + (1 - S) G B_G / K] / D > 0,$$

$$K_G = (F_G / L) [Z_L + S G Z_G / L] / D > 0.$$

Appendix B

The first-order conditions for maximization of (9) corresponding to G, t, T , and w are, respectively,

$$(at + b)L_G + a(TK_G - q) = 0, \quad (\text{B1})$$

$$-V_w + aL + bE_w = 0, \quad (\text{B2})$$

$$(at + b)L_r + a(TK_r + K) = 0, \quad (\text{B3})$$

$$V_w + (at + b)L_w + aTK_w - bE_w = 0. \quad (\text{B4})$$

The Optimal Tax on Capital

Substitute (B2) into (B4) and combine the result with (B3) to obtain,

$$(TK_r + K)(L_w / L_r) = (TK_w + L). \quad (\text{B5})$$

Rearrange this equation to obtain,

$$[-K(L_w / L_r) + L] + T[K_w - K_r(L_w / L_r)] = 0 \quad (\text{B6})$$

Next, using Appendix A's expressions for L_w and L_r , substitute for (L_w / L_r) in the first term in (B6) to obtain,

$$[-(1 - S)F_G G / B_K K] + T[K_w - K_r(L_w / L_r)] = 0 \quad (\text{B7})$$

or

$$[-(1 - S)/K] + T(B_K / F_G G)[K_w - K_r(L_w / L_r)] = 0. \quad (\text{B8})$$

It can be shown that $B_K[K_w - K_r(L_w / L_r)] = 1$. Hence, the optimal tax on capital is

$$T = (1 - S)F_G G / K. \quad (\text{B9})$$

The Optimal Spending Condition

Use the first-order condition (B3) to obtain an expression for $(at + b)$ and substitute it into (B1),

$$-(K + TK_r)(L_G/L_r) + TK_G = q. \tag{B10}$$

Substituting for L_G/L_r from Appendix A yields

$$(K + TK_r)[1 + (1 - S)GB_G/KB_K](F_G/K) + TK_G = q. \tag{B11}$$

Rewrite this last equation as

$$F_G[1 + (1 - S)GB_G/KB_K] + TF_G(K_r/K)[1 + (1 - S)GB_G/KB_K] + TK_G = q. \tag{B12}$$

Next, with some manipulations, it can be shown that

$$TF_G(K_r/K)[1 + (1 - S)GB_G/KB_K] + TK_G = -TB_G/B_K. \tag{B13}$$

Substitute (B13) into (B12) to obtain

$$F_G[1 + (1 - S)GB_G/KB_K] - TB_G/B_K = q. \tag{B14}$$

Finally, substitute the expression for T from (B9) into (B14) to arrive at the spending rule:

$$F_G = q. \tag{B15}$$

The Optimal Tax on Labor

Equations (B9) and (B15) plus the government's budget constraint imply

$$t = SGF_G/L. \tag{B16}$$

Notes

1. Adam Smith recognized the importance of public inputs—"public works" in his terminology—and the difficulty the private sector would have in providing them (see Smith 1937, 681, book V, chap. 1).

2. A third variety of public input is known as firm-augmenting, where there are constant returns to scale in all inputs and the public input is collective to firms themselves. However, a firm can be divided into smaller firms, the effect of which is to create more benefit from the same level of the public input, leading to limitless production possibilities. This unboundedness outcome can be avoided by employing various assumptions (e.g., a minimum size for firms, or a fixed number of firms, or a fixed cost of establishing firms). Nevertheless, Henderson (1974) and McMillan (1979), among others, conclude that it is an implausible specification. For more elaboration, see Feehan (1989).

3. See Diamond and Mirrlees (1971a, 1971b) for the seminal analysis of this efficiency result. This result requires that the government have perfect control over consumer prices and

use of debt policy to maintain the economy on the optimal balanced growth path. If this set of tools is incomplete, then the result no longer goes through. It may also not hold if there are externalities in production, as shown by Batina and Ihori (2005).

4. Judd (1999) recognizes that there is a commons problem associated with a public input of this type but does not study the issue in detail.

5. For example, Aschauer (1989) estimated an aggregate production function using U.S. time series data and included public capital as an input. He found it to be more productive than private capital. Furthermore, he argued that a slowdown in spending on infrastructure could explain the slowdown in productivity growth in the United States in the 1970s. This sparked an extensive literature. However, the evidence on returns to scale is somewhat mixed and it is difficult to draw general conclusions. There are cases of decreasing returns (Batina 1999), providing evidence of the unpaid factors interpretation for public capital. See Batina and Ihori (2005) for a general discussion of the empirical literature on public infrastructure.

6. This requires that the absolute value of F_{GG} be sufficiently small at the commons equilibrium.

7. With a homogeneous production function, the marginal rate of technical substitution between two factors of production is determined solely by the capital-labor ratio. Cost minimization leads firms to choose K and L in a combination such that the marginal rate of substitution equals the ratio of factor prices. The amount of G influences the levels of capital and labor employed but not the cost-minimizing capital-labor ratio.

8. Note that $S = 1/[1 + (r/w)k]$. Firms hire capital and labor such that the marginal rate of substitution coincides with w/r . For homogeneous production functions, that marginal rate of substitution is determined solely by the capital-labor ratio, k . Thus, there is a one-to-one relationship between w/r and k . Denote it as $r/w = h(k)$ and substitute it into the expression for S to obtain $S = 1/[1 + h(k)k]$, so $S = S(k)$.

9. Thus the production function for the public input is simply $G = g(Y) = qY$.

10. Totally differentiate the equilibrium condition and solve to obtain, $\partial w/\partial t = E_w/(E_w - L_w) > 0$, $\partial w/\partial T = L_r/(E_w - L_w) < 0$, $\partial w/\partial i = (L_r - E_i K^+)/ (E_w - L_w)$, and $\partial w/\partial G = L_G/(E_w - L_w)$, where $E_w - L_w > 0$ if supply and demand for labor have their usual slopes.

11. The use of w as a choice variable in the optimization problem is simply a mathematical convenience. Alternatively, one could have used the labor-market clearing condition to express the wage as $w = w(t, T, G)$ and maximized indirect utility subject to the budget constraint alone, with w replaced by $w(t, T, G)$.

12. A side benefit is that this analysis can be readily adapted to a natural common property resource in which there are two primary factors. With no cost of providing such a resource, all the revenues collected from the efficient factor taxes could simply be returned to the households in lump-sum fashion.

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James P. Feehan is a professor of economics and director of the Institute of Social and Economic Research at Memorial University of Newfoundland, Canada. His current research focuses on public investment and fiscal federalism.

Raymond G. Batina received his PhD in economics from the University of Minnesota, and joined the faculty at Washington State University in 1985. His main research and teaching interests are in public policy issues in dynamic general equilibrium models. He is currently working on issues related to tax competition and social security.