Capital-tax financing and scale economies in public-input production

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ABSTRACT

Contrary to the dominant view of inefficient tax competition, Oates and Schwab (1991) show that capital-tax financing of public inputs leads to efficiency when the supply of these inputs is conditioned on business investment (Oates, W.E., Schwab, R.M., 1991. The allocative and distributive implications of local fiscal competition). This paper demonstrates that the cost structure of public-input production is relevant to their proposition on efficient capital-tax financing. That proposition holds if the per-unit cost of public inputs is exogenously fixed; however, it does not hold if public-input production exhibits scale economies. Also, this paper compares our analysis with the Zodrow-Mieszkowski model. That comparison illustrates the importance of the way public inputs are rationed to private firms.

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1. Introduction

There is no doubt that tax competition is now one of the major research subjects in public economics. It is mainly concerned with how competition for mobile tax bases, e.g., capital, among governments affects public spending and taxation policies; see Wilson (1999), Wilson and Wildasin (2004) and Fuest et al. (2005) for excellent surveys of the literature. When analyzing public expenditure, most of the studies on tax competition have focused on public goods that directly benefit residents rather than public inputs that enhance production of private goods. However, public investment in productive activities, including infrastructure, human capital formation and R & D, is an important policy instrument for economic development. It is interesting to examine how public investment is affected when it is financed by source-based taxes on mobile capital.

The standard argument in the literature, which originates with the seminal work of Zodrow and Mieszkowski (1986), is that capital-tax financing causes distortion in capital competition for capital, leading to an inefficient level of public-input provision.¹ In contrast, Oates and Schwab (1991) assert that this provision is efficient even under tax competition. As Oates (1972, p.143) argues, it is not surprising that tax competition distorts public-good provision, because non-benefit taxes are imposed on mobile capital. But, if public inputs are provided, taxes on mobile factors may be benefit taxes that make private firms pay for the services they receive. This “benefit-tax view” of capital taxation, which the present paper calls “the Oates-Schwab efficiency proposition”, is an important basis for the influential argument made by Hulten and Schwab (1997, Section 6) and Oates (1999, Section 3.1; 2001, Section 2) that competition for business investment among governments does not lead to inefficiency.

Oates and Schwab (1991) regard public-input provision as a direct kind subsidy on capital investment, as well as a means of enhancing factor productivities. Specifically, their model assumes that regional governments provide “unpaid factors” to private firms in proportion to those firms’ employment of capital. Public inputs of this type are characterized by the same degree of rivalry as private factors (publicly provided private inputs).² Conditional provision of public inputs is relevant to economic

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See Feehan (1989) for a classification of public inputs. Garcia-Mila and McGuire (2001) and Kellermann (2006) analyze the same rationing scheme for unpaid factors as Oates and Schwab (1991). Feehan and Batina (2007) make a related contribution where unpaid factors are free-access common property. Note that the absence of collective nature in use makes the rationing of unpaid factors possible. Many studies have shown that there are local public services that are subject to a significant degree of congestion (e.g., roads, highways, irrigation systems, garbage collection, and police and fire protection); see Reiter and Weichenrieder (1997) for a review of the related studies.
development policies. For example, Fisher and Peters (1998, p. 42) and Bartik (2004, p.117) report that in the United States, state and local government development programs include firm-specific or site-specific provision of infrastructure as an incentive for new plants or expansions (e.g., public utilities, access roads or bridges). With such ties between investment and public-input provision, private firms will perceive that the marginal gain of investment is equal to the sum of the marginal product of capital plus the return on the rationed public inputs (the public-input rent). The level of capital investment is determined to equate this marginal gain with the gross return (the net return plus the capital tax). In these circumstances, Oates and Schwab (1991) demonstrate that efficient public-input provision is achieved by taxing the return on the rationed inputs away through the marginal product of capital plus the return on the rationed public or expansions (e.g., public utilities, access roads or bridges). With such site-specific rationing mechanism, equilibrium public-input provision and equilibrium regional capital stock will eventually determine the ratio, X/K. Noting that X and K are the aggregate variables in the region, these are exogenous to competitive firms, so that each firm takes X/K as given.5 This rationing scheme for X means that besides the marginal product of capital, the marginal investment creates an additional return equal to R=F(KX): this return on the rationed input is equal to the public-input rent per unit of capital. Thus, the condition for profit-maximizing investment is

\[ F_K + R = r + T, \]  

where \( r \) is the net return on capital, and \( T \) is the (source-based) capital tax rate. This recognition of the influence of the public-input rent on firms' investment decision is an important insight by Oates and Schwab (1991). For later use, the derivatives of \( R \) are provided here:

\[ R_K = \frac{X}{K^2}(F_{KX}K - F_K); \]  

\[ R_X = \frac{1}{K}(F_X + F_{XX}X). \]  

Oates and Schwab (1991) assume that the per-unit cost of the public input is an exogenous constant. We deviate from this assumption by introducing a more general cost structure. To formalize public-input production in a simple manner, it is assumed that the regional public sector produces \( X \) from the numeraire output. (\( C(X) \) units of the numeraire are required to produce \( X \) units of the public input. As usual, scale economies (diseconomies) in production are defined as \( C/X > C \) (\( C/X < C \)).

Under capital-tax financing, the public-budget constraint is

\[ TK = C(X). \]  

The regional government sets public policies to maximize residents' income. Residents are immobile across regions. They own the immobile factor in their region and a certain amount of capital (\( K \)) which may be located in other regions. Thus, residents' income is equal to

\[ \frac{F(L, K, X) - (r + T)K}{r} = F(L, K, X) - (r + T)K. \]  

In Eq. (5), \( F - (r + T)K \) represents non-capital income and \( rK \) is the income from residents' capital endowment. The regional government maximizes Eq. (5) subject to Eqs. (1) and (4). Because the region is small relative to the entire economy, it takes the net capital return as given, which means that \( rK \) is fixed. Therefore, tax and expenditure policies are set to maximize non-capital income.

3 Oates and Schwab (1991) analyze both public goods and public inputs that are financed by a source-based capital tax and a head tax on immobile residents. But, in their model, the head tax is needed only for efficient public-good provision.

4 Zodrow and Mieszkowski (1986) argue that tax competition causes under-provision of public services. But, since Neisit (1995), there have been disputes over whether their under-provision result applies to public inputs; see the studies referred to in Footnote 1.

5 Although the original OS model includes public goods, we omit these goods to concentrate on an analysis of public-input provision (see Footnote 3). A similar remark applies to our comparison with the ZM model (see Section 4).

6 When a private firm employs \( k \) units of capital, the firm perceives that it receives \( X(k) \times k \) units of the public input. In other words, it believes that its share of \( X \) is equal to \( kK \).
3.1. Derivation of the proposition

Our analysis starts with solving the optimization problem formalized in Section 2. The solution to the regional government’s policy problem coincides with the solution to the following Lagrangian function:

\[ F(L, K, X) - (r + T)K + \lambda [F_X(L, K, X) + R(L, K, X) - r - T] + \mu [TK - C(X)] \]

where \( \lambda \) and \( \mu \) are Lagrange multipliers. The exogenous capital income is omitted. Note that in Eq. (6), the condition for profit-maximizing investment, Eq. (1), appears as a constraint of the optimization. Therefore, this formulation requires that \( K \), as well as \( T \) and \( X \), be treated as choice variables; recall that \( L \) is fixed.

The first-order conditions for \( T \) and \( K \) are, respectively,

\[ -K - \lambda + \mu K = 0; \]

\[ -R + \lambda (F_{KX} + R_X) + \mu T = 0. \]

Note that to derive Eq. (8), Eq. (1) was used. Solving Eqs. (7) and (8) for \( \lambda \) and \( \mu \) yields

\[ \lambda = \frac{RK - TK}{\Omega}; \]

\[ \mu = \frac{F_{KK}K + R_K + R}{\Omega} = \frac{F_{KK}K + F_{XX}X}{\Omega} \]

where

\[ \Omega = F_{KK}K + R_K + T = F_{KK}K + F_{XX}X + T - R. \]

Eq. (2) implies the second equality of Eqs. (10) and (11).

The first-order condition for \( X \) is

\[ F_X + \lambda (F_{XX} + R_{XX}) - \mu C = \mu F_X - C + \frac{\lambda}{K} (F_{KK}K - F_X + R_K); \]

\[ = \mu F_X - C + \frac{\lambda}{K} (F_{KK}K - F_{XX}X) = 0 \]

To obtain the first equality in Eq. (12), we used \( \mu - (\lambda/K) = 1 \) from Eq. (7). The second equality was derived by using Eq. (3). Substituting Eqs. (9) and (10) into Eq. (12) yields

\[ (F_{KK}K + F_{XX}X)(F_X - C) = (T - R)(F_{KK}K + F_{XX}X). \]

Also, note from Eq. (4) that \( R - T = (X/K)[F_X - (C/X)] = (X/K)[F_X - C] + (X/K)[C - (C/X)]. \) Substituting this into Eq. (13) and manipulating terms gives the following rule for public-input provision:

\[ Z(F_X - C) = X \left( \frac{C}{X} - C \right) (F_{KK}K + F_{XX}X). \]

where

\[ Z = F_{KK}K + F_{XX}X + \frac{X}{K} (F_{KK}K + F_{XX}X). \]

The linear homogeneity of \( F \) implies that

\[ F_L + F_{Lj} = -F_{Lj}L < 0; Z < 0. \]

where \( (F_{Lj}) = (K, X) \) or \( (X, K) \). It can be seen from Eqs. (14) and (15) that

\[ F_X \geq C \text{ if and only if } \frac{C}{X} \geq C. \]  

In the present model, the marginal product of the public input would be equal to its marginal cost in the first-best optimum: \( F_X = C \). Comparing this first-best rule and Eq. (16) shows that the normative nature of equilibrium public-input provision depends crucially on the nature of the cost function of \( X \). Proposition 1 summarizes the result:

**Proposition 1.** (a) If the per-unit cost of the public input is constant \((C/X = C)\) at any \( X > 0 \), then the level of public-input provision is at its efficient amount under capital-tax financing.

(b) If public-input production exhibits scale economies (diseconomies) in equilibrium, then the level of public-input provision is inefficiently low (high) under capital-tax financing.

Part (a) of Proposition 1 replicates the OS efficiency proposition. Part (b) shows that their proposition does not apply to the case of scale economies or diseconomies. As argued in Section 1, the case of scale economies appears to be more interesting and relevant to our analysis of regional public investment. In what follows, our attention is focused on this case. (The case of scale diseconomies is dealt with in footnotes.)

3.2. Discussion

To explain Proposition 1, it is helpful to derive another expenditure rule, which is expressed in terms of the impact of capital-tax financing of \( X \) on \( K \). The Appendix shows that when, starting from equilibrium, an additional unit of the public input is financed by an increase in the capital tax, the regional capital stock changes by

\[ K(T) = -\frac{F_{KK}K + F_{XX}X}{F_{KK}K + F_{XX}X} X(T). \]

where \( X(T) \) represents the balanced-budget change in \( X \) when \( T \) is increased. It is also shown in the Appendix that \( X(T) > 0 \) under reasonable conditions. This result, together with Eq. (15), implies that \( K(T) < 0 \): capital-tax financing of \( X \) decreases \( K \). Note that this result does not depend on the nature of the cost function of \( X \).

Using Eq. (17), Eq. (13) is transformed into

\[ F_X - C = -\Psi \frac{K(T)}{X(T)}. \]

where

\[ \Psi = T - F_K - r. \]

From (1), \( \Psi \) is equal to the discrepancy between the marginal product of capital and the net return on this factor. \( \Psi \) represents the net marginal benefit of attracting capital from the viewpoint of regional economies: \( F_K \) captures the benefit of increasing capital while \( r \) is the opportunity cost. If \( \Psi > (\leq)0 \), then the regional government would want to increase (decrease) \( K \). From Eqs. (13)–(15), the sign of \( T - R \) is the same as that of \((C/X) - C \) in equilibrium:

\[ \text{sign} \left( \frac{C}{X} - C \right) = \text{sign} \Psi. \]

Eqs. (18) and (19), together with \( K(T) < 0 \), imply that under-provision of \( X \) occurs if \( C/X > C \) because the regional government
curtails public-input provision to prevent capital outflows. On the other hand, if $C/X = C$ as in Oates and Schwab (1991), there is no incentive to change the regional capital stock by manipulating policy instruments. As a result, public-input provision is isolated from the influence of the policy-induced change in the regional capital stock.

With Eq. (19) in mind, we proceed to interpretations of how the cost structure of public-input production affects policy incentives. First of all, note that removing the distortioning incentive for $K$ requires that the rent on the public input be taxed away ($T = R$ or equivalently, $F_X = TK$). When $C/X = C$, this benefit tax finances efficient public expenditure. Indeed, Eq. (4) implies that $F_X = TK = C/X$ and, thus, $F_X = C$. In the OS efficiency proposition, source-based capital taxation plays two different roles at once: it prevents inefficient competition for capital and serves as effective revenue source.

These two roles cannot be met simultaneously when there are scale economies. If $C/X > C$, taxing the public-input rent is not enough to finance efficient expenditure. To see this, suppose hypothetically that the level of $X$ is set such that $F_X = C$. Then, scale economies imply that $F_X = C < C/X$ and, thus, $F_X < C$. In this case, as long as the capital tax rate is set at the level of the efficient benefit tax ($F_X = TK$), there must be an additional non-distorting tax to balance the public budget constraint. Without such a tax, a higher capital tax is required to balance the budget, leading to $F_X < C - TK$ (or $T > R$). This additional levy on capital would cause $\Psi > 0$, resulting in a deviation from the first-best optimum.\(^9\)

### 3.3. Further remarks

The present analysis reveals that source-based capital taxation alone may not yield efficiency in the extended OS model. This argument is akin to García-Mila and McGuire (2001) and Kellermann (2006). García-Mila and McGuire (2001) incorporate external economies of capital investment into the OS model. Internalizing these economies requires that the capital tax rate declines by the marginal external impact of investment. As a result, the capital tax must be complemented by a lump-sum tax to finance efficient public expenditure. Kellermann (2006) extends the OS model to a dynamic framework, showing that as long as competing governments discount future consumption, a lump-sum subsidy and the capital tax must be introduced to sustain efficiency. Thus, these two studies show that although the capital tax is necessary as an efficient benefit tax, other lump-sum taxes or subsidies are also required. By focusing on the cost of public-input production, we have reached a similar conclusion from a quite different angle.

Feehan and Batina (2007) also provide an analysis of unpaid factors that is related to this paper. Their model assumes that once public inputs are provided, these inputs become free-access common property. In this setting, private firms will try to capture the public-input rent by increasing the employment of both immobile and mobile factors while keeping the cost-minimizing factor ratio. This rent-capturing behavior implies that the rent accrues as a return on factor employment: in addition to the marginal product, each factor gets the rent according to its share of regional income. Then, efficiency requires that private factors be taxed to remove the rent accruing to them out of the marginal gain of employment. This benefit tax system is analogous to the OS efficiency proposition, but the taxes must be applied to the immobile factor as well to mobile capital.

### 4. Comparison with the ZM model

Zodrow and Mieszkowski (1986) assume that private firms hire capital up to where the marginal product of capital is equal to the gross return:

$$F_k = r + T. \tag{20}$$

This is distinct from the formulation by Oates and Schwab (1991), as given in Eq. (1). In the presence of unpaid factors that have no collective nature in use, the difference between Eqs. (1) and (20) can be explained in terms of the rationing scheme for publicly provided inputs. As Feehan and Batina (2007) argue, without any rationing system for unpaid factors, the standard marginal conditions for profit maximization (equality of marginal products and factor prices) will not hold as to both immobile and mobile factors, so that Eq. (20) does not apply (see Section 3.3). For Eq. (20) to hold in the presence of unpaid factors, it must be assumed that public-input provision is conditioned on the employment of immobile factors (e.g., firm- or industry-customized job training programs for new workers), rather than capital investment like Eq. (1). This alternate rationing arrangement means that the return on the rationed input is distributed among local residents who are the owners of immobile factors. Since the public-input rent is no longer tied to capital investment, it does not appear in the condition for profit-maximizing investment.\(^10\)

Under these different assumptions about how public-input provision affects investment decisions, there are both similarities and differences between the ZM model and the present one. In Eq. (20), attracting capital is beneficial from the viewpoint of regional economies since $F_X > r$. As we have shown in Section 3.2, the same policy incentive arises in our extended OS model with scale economies in public production. However, the impact of capital-tax financing of public inputs on the regional capital stock may differ between Eqs. (1) and (20). In our model, this financing causes capital outflows under the assumption that the regional production function is CRS in all factors; recall $K(T) < 0$ in Eq. (17). On the other hand, the same assumption on private technology may imply capital inflows when profit maximization is described by Eq. (20). Thus, over-provision of unpaid factors is possible in the ZM model.\(^11\) In our analysis, over-provision is limited to the case of scale diseconomies where capital is driven out; see Footnote 8.) In short, as for public inputs of the unpaid factor type, the direction of expenditure inefficiency depends on whether public-input provision is tied to the employment of immobile or mobile factors.\(^12\)

Another interesting difference between the ZM model and the present one concerns the relevance of the cost structure of public production. This structure does not affect the normative nature of public-input provision in the ZM model.\(^13\) In the present analysis, Eq. (19) implies that depending on the relative magnitude of the average and marginal costs, competing governments have an incentive to increase or decrease capital investment in their territories.

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\(^8\) Over-provision in the case of scale diseconomies is due to the fact that when $\Psi < 0$, public-input provision is used as a means of driving capital out: this argument is similar to "not-in-my-backyard competition" in Markusen et al. (1995).

\(^9\) An analysis of scale diseconomies follows analogously. If the first-best rule for $X$ were met under $C/X < C$, one would have $F_X > C$ because $F_k = C/X$. Thus, when the capital tax is the sole available tax, balancing the public budget means that $F_X > C$, causing $\Psi < 0$. Contrary to the case of scale economies, a lump-sum subsidy must be introduced to keep $T = R$ for efficiency while maintaining the public-budget constraint.

\(^10\) Eq. (20) is also valid as the condition for profit-maximizing investment when public inputs are nonrival or collective in use so that conditional provision is not effective.

\(^11\) Noiset’s (1995, Footnote 4) example of CES production functions can be interpreted such that unpaid factors may be over-provided because the policy-induced capital inflows are possible. (See Matsumoto (1998, Section 3.2) for another interpretation for Noiset (1995) under the assumption that public inputs are collective in use.) As for this point, although Dhillon et al. (2007) argue that CRS in all factors are incompatible with the ZM model, their argument is incorrect because it assumes that capital is the sole private factor.

\(^12\) This argument is related to Matsumoto’s (2004) analysis of factor-specific public inputs where public inputs complementing mobile factors are under-provided while the inputs complementing immobile factors may be over-provided.

\(^13\) A well-known condition for over-provision in the ZM model is that the marginal cost of public inputs is greater than the associated increase in output due to the increased marginal productivity of capital ($C > F_k X$ in terms of the present notation). But, as Matsumoto (1998, Section 3.1) shows, this condition can be transformed into $F_k > F_0 X K$, which has nothing to do with the cost of public production.
On the other hand, once Eq. (20) is assumed, the constraint of capital-tax financing immediately implies that the governments always try to attract the mobile tax base. With this “predetermined” bias towards competition for capital, the cost structure of public production does not play any crucial role in the qualitative analyses of public inputs under tax competition. The relevance of scale economies in this paper is due to the fact that the policy incentive for changing the regional capital stock is endogenously determined.

5. Concluding remarks

The tax competition literature often argues that source-based capital taxation distorts decentralized public policies. Oates and Schwab (1991) challenge this dominant argument in the literature. They demonstrate that when public-input provision is conditioned on capital investment, the capital tax is an efficient benefit tax, rather than a source of economic distortions. Their benefit-tax view of the capital tax can be regarded as one of the pioneering works that finds an efficiency-enhancing role of tax competition.

We have shown that this view is sensitive to how public-input production is modeled. If there are scale economies in production, the OS model returns to the world of Oates (1972) where policy-induced capital outflows result in under-provision of public services. To prevent this “thowback” from occurring, the distorting incentive for attracting capital must be nullified or removed. Provided that introducing a lump-sum tax is infeasible, the studies of tax competition usually emphasize the role of matching or equalization grants in overcoming the distortions due to tax-base mobility (see Wildasin (1989) and Kothenburger (2002)). In our analysis, exploiting scale economies through joint procurements or productions among governments would also reduce the welfare cost of tax competition.

Finally, a remark for future studies might be in order. Following Oates and Schwab (1991), we assumed that the rationing scheme for public inputs is exogenous. An interesting extension is to make this scheme endogenous by considering the case where competing governments choose what private factors or firms are subsidized and how public inputs are supplied to those favored factors or firms. Such analyses will be helpful in understanding both positive and normative aspects of economic development under tax competition.

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Appendix A. Policy-induced change in capital investment

In the present model, Eqs. (1) and (4) yield K and X as functions of T: K(T) and X(T). Differentiating these equations gives

\[ K'(T) = \frac{C - (FKX + R_X)}{\Delta}, \]

\[ X'(T) = \frac{\Omega}{\Delta}, \]

where \( \Delta \equiv C(F_{KK} + R_X) + T(F_{XX} + R_X) \) and the definition of \( \Omega \) is given in Eq. (11). Eq. (A.2) represents the change in X to maintain the public budget constraint while Eq. (A.1) is the impact on K of a balanced-budget change in T and X.

\[ X'(T) = 0 \text{ positive if } (a) \text{ an increase in } T \text{ raises the public budget surplus at a given } X \text{ (the absence of the Laffer effect) and if } (b) \text{ an increase in } X \text{ reduces the surplus at a given } T. \text{ While } (a) \text{ is equivalent to } \Omega < 0, (b) \text{ implies that } \Delta < 0. \text{ As for } (a), \text{ note that Eq. (1)} \text{ yields the change in } K \text{ when } T \text{ is raised at a given } X: \frac{\partial K}{\partial T} = \frac{1}{FKX + R_X}. \text{ Given that } \frac{\partial K}{\partial T} < 0 \text{ (i.e., } F_{KK} + R_X < 0), \text{ one has that } \Omega = F_{KK}K + R_X + T < 0 \text{ when } T \frac{\partial K}{\partial T} + \Delta > 0. \text{ Similarly, differentiating Eq. (1) with respect to } K \text{ and } X \text{ yields that the impact of } X \text{ on } K \text{ at a given } T: \frac{\partial K}{\partial X} = \frac{\Omega - (FKX + R_X)}{FKK + FXX}. \text{ This shows that } \Delta < 0 \text{ when } C' - \frac{T \partial K}{\partial X} < 0. \text{ In what follows, we prove that in equilibrium where Eq. (5) is maximized, Eq. (A.1) is equal to Eq. (17). From Eq. (3), the numerator of Eq. (A.1) is equal to } C' - F_X - (FKK + FXX). \text{ Also, from note Eq. (12) that } C' - F_X = (\lambda \mu K) (FKX + FXX) \text{ in equilibrium. Thus, Eq. (A.1) is reduced to}

\[ K'(T) = \frac{(\lambda / \mu K - 1)(FKX + FXX)}{\Delta}, \]

Eqs. (7) and (10) imply that \( (\lambda \mu K - 1) = -1/\mu = -\Omega/(FKK + FXX) \text{ in equilibrium. Substituting this into Eq. (A.3) gives}

\[ K'(T) = -\frac{FKX + FXX}{FKK + FXX} \Omega \]

This, together with Eq. (A.2), completes the proof.

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\[ Eq. (11)\]

\[ Eq. (A.2)\]

\[ \lambda \mu K - 1 = -1/\mu = -\Omega/(FKK + FXX) \]

\[ K'(T) = \frac{(\lambda / \mu K - 1)(FKX + FXX)}{\Delta}, \]

\[ K'(T) = -\frac{FKX + FXX}{FKK + FXX} \Omega \]