

ECONOMICS 6002 CLASSES 10-11
NON-LINEAR ESTIMATION

1. Nonlinearity in Econometric Models
 - a. Role of the assumption of linearity in the Classical Linear Regression Model
 - i. Non-linearities preserve asymptotic properties (consistency, asymptotic normality, etc.) but NOT (in general) small-sample statistical properties (normality, unbiasedness, variance estimation)
 - b. **Linearity in parameters** (even with non-linear variables) preserves small-sample properties
 - c. **Intrinsic linearity** preserves asymptotic properties but not small sample properties. They can however be estimated using linear methods.
Example: Cobb-Douglas production function.
 - d. **Intrinsic non-linearities** preserve asymptotic properties but not small sample properties. They cannot be estimated using linear methods.

2. Rationale for non-linear estimation
 - a. Better fit with the data
 - b. Better theoretical implications
 - c. Some issues are inherently non-linear

3. Mathematical Solution to Non-Linear Least Squares estimator
 - a.
$$\frac{\partial \sum \hat{\varepsilon}_t^2}{\partial \hat{\beta}_j} = 2 \sum (y_t - h(x_t, \hat{\beta})) \left(- \frac{\partial h(x_t, \hat{\beta})}{\partial \hat{\beta}_j} \right) = 0$$
 - b. Compare to OLS
$$\frac{\partial \sum \hat{\varepsilon}_t^2}{\partial \hat{\beta}_j} = 2 \sum_t (y_t - \sum_k \hat{\beta}_k x_{kt}) (-x_{jt}) = 0$$

4. Role of the **Linearized Regression**
 - a. A linear approximation to the non-linear regression:
 - i. Determines the asymptotic properties of the NLS estimator
 - ii. Can be used to estimate values of the NLS estimator

5. The Gauss-Newton solution algorithm
 - a. Based on the linearized regression

6. Role of the **pseudoregressors**
 - a. These play the same role in the non-linear model as do the regressors in the linear model, but are functions of the data and the estimates and not just the data themselves
 - b. Pseudoregressors act as regressors in the Linearized Regression
 - c. The (asymptotic) covariance matrix of the NLS estimator depends on the pseudoregressors

7. Estimation difficulties:
 - a. Non-convergence
 - b. Multiple solutions
 - c. Multicollinearity

8. Non-linear hypothesis testing
 - a. t and F tests no longer exact, because estimates are non-linear functions
 - b. Wald test is applicable
 - i. “t-test” is a variant of the Wald test
 - c. The Lagrange Multiplier test
 - i. Based on the **restricted** estimates
 - ii. Three variants
 - (1) Test on Lagrange multipliers
 - (2) Test on gradient vector of unrestricted model (but at restricted model estimates)
 - (3) Test on linear regression of residuals on pseudoregressors of unrestricted model (but at restricted model estimates)
 - d. An F-type test is also available
 - e. Relationship between tests

9. Alternative non-linear estimation solution methods
 - a. Back-and-forth
 - b. Grid search
 - i. Box-Cox transformation

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